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Propagation of Gaussian beams in negative-index metamaterials with cubic nonlinearity

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Abstract

Propagation of Gaussian beams in the negative-index metamaterials (NIMs) with cubic nonlinearities is investigated, both theoretically and numerically. The role of the status of the incident Gaussian beam, which is scaled by a converging parameter in this paper, in beam self-focusing and self-defocusing in NIMs is specially identified. The expressions for beam self-focusing and self-defocusing for different converging parameter cases, and the dependence of the critical power and the focus location of self-focusing in NIMs on the converging parameter are obtained. It is found that it is the divergent rather than convergent incident beams which are self-focused more quickly in NIMs with defocusing nonlinearities, in sharp contrast with the propagation property of Gaussian beams in conventional Kerr media, in which beam self-focusing only occurs in the media with focusing nonlinearities and a convergent incident beam self-focuses more quickly than a divergent one. By adjusting the converging parameter of incident Gaussian beam or the controllable magnetic permeability of NIM, or both, one can manipulate the beam self-focusing in NIMs at will.

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Keywords: Nonlinear optics; Negative-index metamaterials; Gaussian beam; Self-focusing; Self-defocusing

1. Introduction

Recently, researchers have devoted a great deal of attention to negative-index metamaterials (NIMs), artificial materials with simultaneously negative electric permittivity and magnetic permeability, due to their unique properties and exciting applications [1]. NIMs were first obtained in the microwave range [2]. Now negative-index up to the optical range can be realized [1]. Moreover, the inclusion of nonlinear elements within metamaterials, for instance by adding diodes in the split-ring resonators' paths, can induce quadratic [3] or cubic [4] nonlinear responses. This provides us a chance to reconsider the classical linear and nonlinear processes by exploiting the unusual and sometimes counterintuitive properties of NIMs.

In virtual of the developments in NIMs, the propagation of electromagnetic wave in NIMs was studied analytically and numerically in many works. In the linear regime, taking the propagation of Gaussian beams in NIMs for example, it has been found that there is a unique lateral displacement when a Gaussian beam is reflected from or transmitted through a slab of NIM with an angle of incidence [5–8], and that the phase difference caused by the Gouy phase shift in conventional medium can be compensated by the one caused by the inverse Gouy phase shift in NIM [9]. For describing linear propagation in NIMs, partial differential equation is derived and transfer functions are developed [10]. In the nonlinear regime, there also have some works on second-order (for review, see, e.g., [1]) and third-order [11–20] nonlinear optical phenomena in NIMs. For the propagation of Gaussian beams in media with

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cubic nonlinearity, the interplay between diffraction and nonlinear effect can result in beam self-focusing or self-trapping, also called spatial soliton [21]. The spatial solitons are found supported [14,15,22] and the stabilization of spatial solitary waves can be obtained by the nonlinearity or diffraction management achieved through a periodic variation of the nonlinearity or diffraction coefficient [22]. Moreover, surface-polariton solitons can exist at the interface between a nonlinear NIM and a linear conventional medium [12]. Because of the unique electromagnetic properties of NIMs, these physical phenomena have different features from their counterparts in conventional Kerr medium cases. Besides, in the negative refraction region of a photonic crystal with a positive Kerr coefficient, the self-focusing effect is found completely suppressed [23]. An important potential application of NIMs with cubic nonlinearity (Kerr NIMs) is that they can compensate small-scale self-focusing or filamentation [24,25] in conventional Kerr media [13]. These unprecedented propagation properties show that NIMs provides unique opportunities for developing new approaches to manipulate electromagnetic waves. It should be pointed out that nonlinear optics in NIMs remains so far a less developed branch of optics. As a typical nonlinear optical phenomenon, beam self-focusing in nonlinear NIMs also deserves a systematic reconsideration. On the other hand, it is well known that beam propagation in nonlinear medium is closely related to the property of incident beam. The main purpose of this paper is to investigate the nonlinear propagation of Gaussian beams in Kerr NIMs and to identify the influence of the focusing property of incident beams on it.

This paper is organized as follows. In Section 2, we model the beam propagation in Kerr NIMs. In Section 3, we derive the expression for the central intensity of Gaussian beam in Kerr NIMs and investigate the influence of the focusing property of incident Gaussian beam on self-focusing and self-defocusing. The role of the magnetic permeability in controlling beam self-focusing and self-defocusing is also demonstrated. In Section 4, we present numerical simulations to verify the theoretical results. Section 5 concludes the paper.

2. Modeling the incident Gaussian beam and the propagation of light beam in Kerr NIMs

In practice, beam propagation in a medium can be strongly influenced by the beam status before the medium. We assume that the incident beam is a paraxial ground-mode Gaussian beam, whose electric field is a linearly polarized field along x axis, as shown in Fig. 1. The optical axis is perpendicular to the surface of the Kerr NIM. In this paper, we assume that the NIM is homogeneous, isotropic and lossless.

The free space before the NIM in Fig. 1 can be viewed as a positive-index medium (PIM). Then, the status of the Gaussian beam before it enters into the NIM is described by

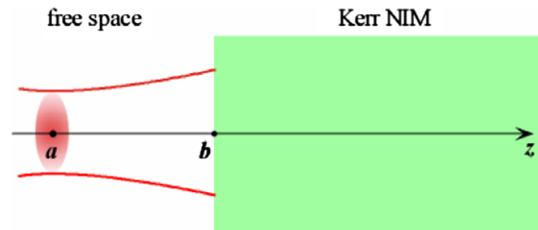

Fig. 1. Sketch map for the incident beam and the on-axis coordinate of the Kerr NIM. The waist of the Gaussian beam is located at $z = a$, and the NIM fills the half space of $z > b$.

$$E(x,y,z) = C\frac{w_0}{w(z)}\exp\left(-\frac{x^2+y^2}{w(z)^2}\right) \times \exp\left\{-\mathrm{i}\left[k_0(z-a)+k_0\frac{x^2+y^2}{2R(z)}-\Delta\phi(z)\right]\right\}, \tag{1}$$

where C and w_0 are the peak amplitude and the spot radius at the beam waist at $z = a$, respectively, k_0 is the wave number in free space and equals to ω/c, ω is the angular frequency. The beam spot radius, the radius of the curvature of the wave front and the Gouy phase in Eq. (1) are

$$\begin{aligned} w(z) &= w_0\sqrt{1+\left(\frac{z-a}{L_\mathrm{R}}\right)^2}, \\ R(z) &= (z-a)\left[1+\left(\frac{L_\mathrm{R}}{z-a}\right)^2\right], \\ \Delta\phi(z) &= \arctan\left(\frac{z-a}{L_\mathrm{R}}\right), \end{aligned} \tag{2}$$

respectively, where $L_\mathrm{R} = k_0 w_0^2/2$ is the Rayleigh length in free space. Here we have assumed that the absolute phase of the incident beam is zero at $z = a$.

To model the nonlinear propagation of light beam in NIMs we start with the following Maxwell equations:

$$\begin{cases} \nabla\times\mathbf{E} = -\mathrm{i}\omega\mathbf{B}, \\ \nabla\times\mathbf{H} = \mathrm{i}\omega\mathbf{D}, \\ \nabla\cdot\mathbf{D} = 0, \\ \nabla\cdot\mathbf{B} = 0, \end{cases} \tag{3}$$

where $\mathbf{E}$ and $\mathbf{H}$ are electric and magnetic fields, respectively, and $\mathbf{D}$ and $\mathbf{B}$ are electric and magnetic flux densities which arise in response to the electric and magnetic fields inside the medium and are related to them through the constitutive relations given by $\mathbf{D} = \varepsilon\mathbf{E} + \mathbf{P}_\mathrm{NL}$ and $\mathbf{B} = \mu\mathbf{H}$, respectively, $\varepsilon = \varepsilon_\mathrm{r}\varepsilon_0$ is effective electric permittivity and $\mu = \mu_\mathrm{r}\mu_0$ effective magnetic permeability, ε_r and μ_r are relative permittivity and relative permeability, respectively, $\mathbf{P}_\mathrm{NL} = \varepsilon_0\chi_\mathrm{p}|\mathbf{E}|^2\mathbf{E}$ is the nonlinear parolization, and χ_p is the third-order nonlinear susceptibility. Applying $\nabla\times$ to the first and the second equations of the set (3), respectively, and considering the beam propagates along the z direction, we can get the propagation equation for electric field in the form