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Goos–Hänchen shift of the uniaxially anisotropic left-handed material film with an arbitrary angle between the optical axis and the interface

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Abstract

The Goos–Hänchen shift at the surface of a uniaxially anisotropic left-handed material film is investigated, for the situation of that there is an arbitrary angle between the optical axis and the interface of the material. The analytical expressions of the Goos–Hänchen shifts are derived, for both cases of that the total reflection occurs and does not occur at the first interface. The sign of the Goos–Hänchen shifts in the two situations is analysed. The results show that the Goos–Hänchen shift of the reflected wave is the same as that of the transmitted one for the case that the total reflection does not occur at the first interface; the Goos–Hänchen shift of the transmitted wave oscillates as the thickness of the film is increased, and its overall tendency is increased; the Goos–Hänchen shift of the transmitted wave realizes its absolute maximum when the transmitted wave resonances, the absolute maximum is almost several 10 times of the wavelength of the incident wave; the Goos–Hänchen shift of the transmitted wave is significantly influenced by the incident angle and the angle between the optical axis and the interface.

Keywords: Uniaxially anisotropic; Left-handed material; Goos–Hänchen shift; Total reflection

1. Introduction

Recently, there has been great interest in a new type of electromagnetic materials called left-handed materials [1,2], which have many special characteristics such as negative refractive indices, reversed Doppler effect, reversed Cerenkov effect, evanescent waves amplifying, negative Goos–Hänchen (GH) shift etc., because the materials have simultaneously negative permittivities and permeabilities. The electromagnetic characteristics of uniaxially anisotropic left-handed material were investigated [3]. It was found that the properties of anisotropic left-handed materials were greatly different to those of isotropic left-handed materials; for uniaxially anisotropic left-handed materials it was not necessary that all the elements of the permittivity and permeability tensors were negative [3]. The analytical expressions of the GH shift at the surface of the isotropic right handed material and uniaxially anisotropic left-handed material were given [4,5] for the cases of that the optical axis was normal or parallel to the interface. It is pity that the papers mentioned above are limited to the only cases of that optical axis was normal or parallel to the interface. The GH shift of occurring at the two interfaces of uniaxially anisotropic left-handed material film with an arbitrary angle between the optical axis and the interfaces is investigated, the sign of the GH shift is discussed, the influence of the incident angle and the angle between the optical axis and the interface is analysed in this paper.

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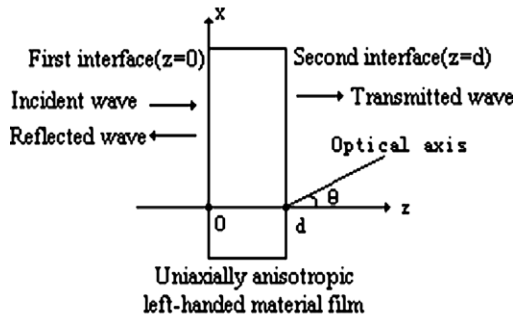

Fig. 1. Schematic diagram of uniaxially anisotropic left-handed material film.

2. The dispersion equation of the uniaxially anisotropic left-handed material and the condition to ensure the material to be left-handed

The uniaxially anisotropic left-handed material film is shown in Fig. 1. The thickness of the film is denoted as d. The material stuck on the both sides of the film is isotropic right handed material, whose permittivity and permeability are denoted as ε_r and μ_r, respectively.

Let the x–y plane be parallel to the first interface and the y axis be normal to the optical axis. The angle between the optical axis and z axis is denoted as θ. The permittivity and permeability of the uniaxially anisotropic left-handed material have the forms according to Ref. [6]

$$\varepsilon = \begin{bmatrix} \varepsilon_\perp - \Delta\varepsilon \sin^2\theta & 0 & \Delta\varepsilon \sin\theta\cos\theta \\ 0 & \varepsilon_\perp & 0 \\ \Delta\varepsilon\sin\theta\cos\theta & 0 & \varepsilon_\perp - \Delta\varepsilon\cos^2\theta \end{bmatrix} \tag{1}$$

$$\mu = \begin{bmatrix} \mu_\perp - \Delta\mu \sin^2\theta & 0 & \Delta\mu \sin\theta\cos\theta \\ 0 & \mu_\perp & 0 \\ \Delta\mu\sin\theta\cos\theta & 0 & \mu_\perp - \Delta\mu\cos^2\theta \end{bmatrix} \tag{2}$$

where $\Delta\mu = \mu_\perp - \mu_{//}$, $\Delta\varepsilon = \varepsilon_\perp - \varepsilon_{//}$, $0 < \theta < \frac{\pi}{2}$.

Only TE waves are investigated here which can be expressed as

$$E = E_0 e_y \exp(\mathrm{i}k_x x + \mathrm{i}k_{Ez} z - \mathrm{i}\omega t) \tag{3}$$

From Eq. (3) and Maxwell equations, one can get

$$H = \frac{E_0}{\omega}(-Ae_x + Be_z)\exp(\mathrm{i}k_x x + \mathrm{i}k_{Ez} z - \mathrm{i}\omega t) \tag{4}$$

where e_x, e_y, e_z are the unit vectors along the x, y, and z axes; k_x and k_{Ez} are the x component and z component of the wave vector; A and B are expressed as

$$A = \frac{(\mu_\perp - \Delta\mu\cos^2\theta)k_{Ez} + \Delta\mu k_x \sin\theta\cos\theta}{\mu_\perp\mu_{//}},$$

$$B = \frac{(\mu_\perp - \Delta\mu\sin^2\theta)k_x + \Delta\mu k_{Ez}\sin\theta\cos\theta}{\mu_\perp\mu_{//}}$$

From Eqs. (3), (4) and Maxwell equations, the dispersion equation is obtained

$$\frac{k_x^2 + k_{Ez}^2}{\mu_{//}} - \frac{\Delta\mu(k_x\sin\theta - k_{Ez}\cos\theta)^2}{\mu_\perp\mu_{//}} = \omega^2\varepsilon_\perp \tag{5}$$

From Eqs. (3)–(5), the inner product of the time-averaged Poynting vector of the incident wave and the wave vector $\langle S\rangle \cdot k_E = \frac{1}{2}\omega\varepsilon_\perp E_0^2$ can be calculated. In order to ensure the material is left-handed, the $\varepsilon_\perp$ must be negative. This result is the same to the conclusion for normally incident case [3]. Therefore, the change of the angle between the optical axis and the interface of the material does not change the left-handed characteristics of the material.

3. The GH shift at the first interface when the incident waves are totally reflected

When total reflection occurs at the first interface, the incident wave and the reflected wave are expressed as

$$E_\mathrm{i} = E_0 e_y \exp(\mathrm{i}k_x x + \mathrm{i}k_z z - \mathrm{i}\omega t) \tag{6}$$

$$E_r = rE_0 e_y \exp(\mathrm{i}k_x - \mathrm{i}k_z z - \mathrm{i}\omega t) \tag{7}$$

where r is the overall reflection coefficient of the film when the total reflection occurs at first interface; k_z is z component of the wave vector of the wave in the isotropic right handed materials, which satisfying the dispersion relation

$$k_x^2 + k_z^2 = k^2 = \omega^2\varepsilon_r\mu_r \tag{8}$$

The reflection coefficient can be obtained from Eqs. (6), (7), Maxwell equations and the boundary conditions, from which one has

$$r = \frac{k_z - A\mu_r}{k_z + A\mu_r} \tag{9}$$

The reflection coefficient can also be expressed as

$$r = |r|\exp(-\mathrm{i}\phi) \tag{10}$$

For the total reflection situations, the z component of the wave vector is imaginary. From Eqs. (5) and (8), it can be obtained

$$k_{Ez} = \frac{-\sin\theta\cos\theta\Delta\mu k_x}{\mu_\perp - \Delta\mu\cos^2\theta} + \mathrm{i}\sigma\sqrt{\frac{X}{(\mu_\perp - \Delta\mu\cos^2\theta)^2}} \tag{11}$$

According to the principle of energy conservation, it can be seen that $\sigma = 1$. The X satisfies the relation of

$$X = [k^2\sin^2\alpha - \omega^2(\mu_\perp - \Delta\mu\cos^2\theta)\varepsilon_\perp]\mu_\perp\mu_{//} \geqslant 0 \tag{12}$$

where α is the incident angle.

If $(\mu_\perp - \Delta\mu\cos^2\theta) \leqslant 0$, a critical angle can be gotten from Eq. (12), which is expressed as

$$\alpha_\mathrm{c} = \arcsin\sqrt{\frac{(\mu_\perp - \Delta\mu\cos^2\theta)\varepsilon_\perp}{\varepsilon_r\mu_r}} \tag{13}$$

3.1. Discussion on the range of the total reflection angle and the sign of the GH shift

(1) If the $\mu_\perp\mu_{//}$ and $(\mu_\perp - \Delta\mu\cos^2\theta)$ are negative, from Eqs. (9) and (10) one has