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On the origins of decoherence and extinction contrast in phase-contrast imaging

Ya.I. Nesterets *

CSIRO, Materials Science and Engineering, PB33, Clayton South, Vic. 3169, Australia

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Abstract

A theoretical formalism describing the formation of images in a linear shift invariant X-ray optical system is derived within the wave-optical theory. It is applicable to a non-crystalline object consisting of two types of features, with the characteristic sizes which are respectively not smaller and much smaller than the resolution of the imaging system. This formalism is then applied to two phase-contrast imaging techniques, the propagation-based and analyser-based imaging. The obtained formulae for the intensity distribution in the image well explain the "decoherence effect" which is observed in the former technique and the "extinction contrast" which is a characteristic of the latter technique. This formalism is shown to be in good agreement with the results of the accurate numerical simulations, using rigorous wave-optical theory, of the propagation-based and analyser-based phase-contrast images of the model objects.

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1. Introduction

X-ray phase-contrast imaging techniques are routinely employed for studies of biological and non-biological objects. In general, each object contains features of different sizes, including those whose characteristic size is much larger than the resolution of the imaging system, as well as the features that are unresolvable by the imaging system. Although being unresolvable, these fine features significantly affect the phase-contrast images, in a way specific for each phase-contrast imaging technique. For example, in propagation-based imaging (PBI) [1–3] the fine features can result in appearance of speckles (see, for example, [4] and references herein) as well as in significant reduction of contrast of the resolvable features if those are overlapped by the fine features. The latter effect is usually attributed to the loss of coherence in the X-ray beam ("decoherence effect") that results in appearance of a low coherence component in the beam [5]. In fact, as was emphasized by Nugent and colleagues [6], it is the finite resolution of the experimental imaging system that performs the spatial ensemble average that only appears as a loss of coherence (the degree of coherence is actually preserved while the radiation propagates through the system). In analyser-based imaging (ABI) [7–9], similar effects are also observed and the most profound one is the so-called "extinction contrast" (see, for example, [10–12]) which originates from the finite angular width of the analyser reflectivity curve which filters out the high angular components of the incident wave.

Given a complex amplitude of the wave in the exit plane of an object, the corresponding intensity distribution in the detector plane can be accurately calculated in PBI and ABI using wave-optical formalism (direct problem, see Section 2). Unfortunately, the inverse problem, consisting in reconstruction of the complex amplitude of the object wave given an intensity distribution

* Tel.: +61 3 95452916; fax: +61 3 95441128.
E-mail address: Yakov.Nesterets@csiro.au.

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in the corresponding image, has no general analytical solution within the wave-optical approach. However, under certain conditions on the properties of the object wave or the optical system, an approximate solution of the direct problem can be obtained that could be easily inverted to give a solution of the inverse problem. For example, if the complex amplitude of the object wave varies slowly on a length scale, equal to the first Fresnel zone width in PBI and to the extinction length of the crystal in ABI, the transport of intensity equation (TIE) [13] and the geometrical optics approximation (GOA) [14,15] are valid in PBI and ABI respectively. The GOA is widely used in ABI. For example, a well-known DEI phase/amplitude reconstruction algorithm [16] is based on the GOA. Note however, that the validity conditions of the GOA are strongly violated in the presence of fine features in the object. In order to take the "extinction effect" into account, two modifications to the main formula of the GOA [14,15] were proposed, respectively in [10] and [12,17] under assumption that fine features are unresolvable by the imaging system. Note however, that both modifications were presented without a rigorous justification, using simple semi-empirical considerations.

In this paper, using rigorous wave-optical formalism, we derive a general formula for an intensity distribution in the image produced by an arbitrary linear shift invariant imaging system. This formula is applicable to a phase object that consist of two types of features: large features which are well resolvable by the system; and fine features which are much smaller than the large ones and are unresolvable by the imaging system. This result is consequently applied in Section 3 to the PBI and ABI techniques.

2. Main assumptions and general formulae

Let *a monochromatic plane wave* of unit intensity, $\exp(\mathrm{i}kz)$, be incident on *a pure phase object*. Here $k = 2\pi/\lambda$ is the wave number in vacuum corresponding to the X-ray wavelength λ. We assume that *the projection approximation is valid* and therefore the propagation of this wave through the object can be described by a transmission function of the object, $q(x) \equiv \exp[\mathrm{i}\varphi(x)]$, so that the complex amplitude of the wave in the exit plane of the object is simply a product $\exp(\mathrm{i}kz)\ q(x)$. The intensity of this wave is one; which indicates that no information about the object can be extracted from this contact image. In order to visualize the phase $\varphi(x)$ induced by the object, *a linear shift-invariant* (LSI) optical system can be installed between the object and the detector. Any LSI system is characterised in the real space by a complex propagator $P(x)$ that relates the complex amplitude $E(x)$ of the wave on the exit of the system (in the detector plane) to the complex amplitude $\exp(\mathrm{i}kz)\ q(x)$ of the wave on the entrance of the system as follows:

$$E(x) = \exp(\mathrm{i}kz)\int \mathrm{d}x' P(x')q(x-x'). \tag{1}$$

The intensity distribution in the detector plane is then written as follows:

$$\begin{aligned} I(x) &= \int\int \mathrm{d}x'\mathrm{d}x'' P(x')P^*(x'')q(x-x')q^*(x-x'') \\ &= \int\int \mathrm{d}x'\mathrm{d}x'' P(x')P^*(x'') \\ &\quad \times \exp\left[\mathrm{i}\{\varphi(x-x') - \varphi(x-x'')\}\right]. \end{aligned} \tag{2}$$

Taking into account finite resolution of the imaging system (due to the finite source size and detector resolution), the detected image $\widetilde{I}(x)$ is obtained by convolution of the intensity $I(x)$ (that would be obtained by an ideal system with delta-like resolution) and the point spread function (PSF) $S(x)$ of the imaging system:

$$\widetilde{I}(x) = \int \mathrm{d}y I(x-y)S(y). \tag{3}$$

It is worth summarising the assumptions made so far:

(1) monochromatic incident plane wave of unit amplitude;
(2) pure phase object;
(3) validity of the projection approximation;
(4) LSI imaging system.

We now turn our attention to the object and introduce two practically important models.

2.1. An object consisting of randomly distributed unresolvable features

Let an object consist of small features that are not resolvable by the system (solely due to the finite resolution of the detector in our idealised case of a monochromatic plane incident wave). Then the phase of the wave transmitted through the object is fast oscillating on the length scale of the resolution of the system, which will be indicated by an index f in the phase function, $\varphi_f(x)$, and the corresponding intensity in the image, $I_f(x)$. Using Eqs. (2) and (3), the latter is written as follows:

$$\begin{aligned} \widetilde{I}_f(x) = \int\int \mathrm{d}x'\mathrm{d}x'' P(x')P^*(x'') \int \mathrm{d}y S(y) \\ \times \exp\left[\mathrm{i}\left\{\varphi_f(x-y-x') - \varphi_f(x-y-x'')\right\}\right]. \end{aligned} \tag{4}$$

If we assume that these unresolvable features are distributed randomly and the characteristic size of the features is much smaller than the width of the system PSF, $S(x)$, then the inner integral in Eq. (4) can be well approximated as follows:

$$\begin{aligned} &\int \mathrm{d}y S(y) \exp\left[\mathrm{i}\left\{\varphi_f(x-y-x') - \varphi_f(x-y-x'')\right\}\right] \\ &\cong \int \mathrm{d}y S(y)\Gamma(x'-x''; x-y), \end{aligned} \tag{5}$$

where $\Gamma(x'-x''; x) \equiv \overline{\exp\left[\mathrm{i}\left\{\varphi_f(x-x') - \varphi_f(x-x'')\right\}\right]}$ is an autocorrelation function of the random phase induced by