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# Propagation characteristics of higher-order annular Gaussian beams in atmospheric turbulence

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**Abstract**

The propagation characteristics of higher-order annular Gaussian (HOAG) beams in turbulence are investigated. From a HOAG source plane excitation, the average intensity of the receiver plane is developed analytically. This formulation is verified against the previously derived HOAG beam solution in free space. The graphical outputs indicate that, upon traveling in turbulent atmosphere, the HOAG beam will undergo different stages of evolution. At intermediate propagation distances, it will attempt to concentrate the energy near the origin. In this process, the appearance of the single higher-order primary beam will be encountered. Eventually HOAG originated beam will become a pure Gaussian beam after propagating an excessive distance in the turbulent medium.


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## 1. Introduction

In two of our recent works, we have presented the propagation of higher-order annular beams in free space as well as their log-amplitude and phase fluctuations in the presence of atmospheric turbulence [1,2]. With these studies, our primary aim was to examine the suitability of various beams for broadband access free space optics (FSO) communication links. The present work is a continuation of such efforts in which the intensity at the source plane and the average intensity at the receiver plane of HOAG beams are evaluated against the variations in the source and propagation parameters, namely respective source sizes, amplitude factors, displacement parameters, mode indices, propagation distance, wavelength of operation and turbulence strength.

There is some literature on the subject of annular beams primarily dealing with its production at the resonator stage [3–10]. In a recent publication, Vetelino and Andrews [11] demonstrated that a doughnut-shaped annular Gaussian beam may have favourable scintillation properties as compared to a collimated Gaussian beam without causing additional beam spreading or irradiance loss for long propagation distances in atmospheric turbulence. To our knowledge, the propagation characteristics of higher-order annular laser beam in turbulence has not been investigated so far.

The HOAG beams do not necessarily retain the doughnut shape as formed by the fundamental Gaussian ones. However, we continue to use the term "annular" to be in line with the present terminology.

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## 2. Formulation of source and receiver plane intensities for HOAG beams

Conventionally the annular beam comprises from the difference of two co-centric beam fields, called primary and the secondary. Hence a HOAG beam aligned to be at the origin of the source plane oriented to be perpendicular to the axis of propagation, $z$, will have the field distribution

$$\begin{aligned} u_s(s_x, s_y, z=0) = {} & A_{n1m1} H_{n1}(a_{x1}s_x + b_{x1}) H_{m1}(a_{y1}s_y + b_{y1}) \\ & \times \exp[-0.5k(\alpha_{x1}s_x^2 + \alpha_{y1}s_y^2)] \\ & - A_{n2m2} H_{n2}(a_{x2}s_x + b_{x2}) H_{m2}(a_{y2}s_y + b_{y2}) \\ & \times \exp[-0.5k(\alpha_{x2}s_x^2 + \alpha_{y2}s_y^2)], \end{aligned} \quad (1)$$

where $s_x$ and $s_y$ are the $x$ and $y$ components of the source plane vector $\mathbf{s}$, such that $\mathbf{s} = (s_x, s_y)$. In Eq. (1), the index numeral 1 refers to the primary beam, while 2 stands for the secondary beam. $A_{n1m1}$ is the amplitude factor of the primary field. $H_{n1}(a_{x1}s_x + b_{x1})$ and $H_{m1}(a_{y1}s_y + b_{y1})$ are Hermite polynomials describing the beam properties for $s_x$ and $s_y$ directions, where $n_1$ and $m_1$ are the order, $a_{x1}$ and $a_{y1}$ characterize the width, $b_{x1}$ and $b_{y1}$ are the complex displacement parameters,

$$\alpha_{x1} = 1/(k\alpha_{sx1}^2) + i/F_{x1}, \quad \alpha_{y1} = 1/(k\alpha_{sy1}^2) + i/F_{y1}, \quad (2)$$

where $\alpha_{sx1}$, $\alpha_{sy1}$ are Gaussian source sizes, and $F_{x1}$, $F_{y1}$ are the focusing parameters along $s_x$ and $s_y$ directions, $k$ is the wave number and $i = (-1)^{1/2}$. Equivalent definitions will apply to the parameters of the secondary beam, by changing the subscript index from 1 to 2.

The source beam intensity of the HOAG beam is obtained via multiplying the field expression, i.e., Eq. (1), by its complex conjugate, thus

$$I_s(\mathbf{s}, z=0) = I_s(s_x, s_y, z=0) = u_s(s_x, s_y, z=0)u_s^*(s_x, s_y, z=0), \quad (3)$$

where $*$ denotes the complex conjugate.

The field, $u_r(\mathbf{p}, L, t)$, arriving on a receiver plane, assumed to be located perpendicular to the axis of propagation at $z = L$, is found via Huygens–Fresnel integral as

$$\begin{aligned} u_r(\mathbf{p}, L, t) = {} & [k\exp(ikL)/(2i\pi L)] \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{d}^2\mathbf{s}\, u_s(s_x, s_y, z=0) \\ & \times \exp[ik(\mathbf{p}-\mathbf{s})^2/(2L) + \psi(\mathbf{s}, \mathbf{p}) - 2i\pi f t], \end{aligned} \quad (4)$$

where $\mathbf{p} = (p_x, p_y)$ represents the transverse receiver coordinate, $u_s(s_x, s_y, z=0)$ is the field of the HOAG beam at the source plane as supplied in Eq. (1), $\psi(\mathbf{s}, \mathbf{p})$ is the solution to Rytov method representing the random part of the complex phase of a spherical wave propagating from the source point $(\mathbf{s}, z=0)$ to the receiver point $(\mathbf{p}, z=L)$, $f$ is the frequency, and $t$ refers to time.

Due to the existence of turbulence along the axis of propagation, we are naturally interested in the quantities averaged over the medium statistics. This way, the average intensity at the receiver plane will become $\langle I_r(\mathbf{p}, L)\rangle = \langle u_r(\mathbf{p}, L, t)u_r^*(\mathbf{p}, L, t)\rangle$, where $\langle\,\rangle$ stands for the ensemble averaging. Using Eq. (4), $\langle I_r(\mathbf{p}, L)\rangle$ turns into

$$\begin{aligned} \langle I_r(\mathbf{p}, L)\rangle = {} & [k^2/(2\pi L)^2] \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{d}^2\mathbf{s}_1 \mathbf{d}^2\mathbf{s}_2 \\ & \times u_s(s_x, s_y, z=0)u_s^*(s_x, s_y, z=0) \\ & \times \exp\{ik[(\mathbf{p}-\mathbf{s}_1)^2 - (\mathbf{p}-\mathbf{s}_2)^2]/(2L)\} \\ & \times \langle\exp[\psi(\mathbf{s}_1, \mathbf{p}) + \psi^*(\mathbf{s}_2, \mathbf{p})]\rangle, \end{aligned} \quad (5)$$

The ensemble average term appearing on the fourth line of Eq. (5) is [12]

$$\begin{aligned} \langle\exp[\psi(\mathbf{s}_1, \mathbf{p}) + \psi^*(\mathbf{s}_2, \mathbf{p})]\rangle & = \exp[-0.5D_\psi(\mathbf{s}_1 - \mathbf{s}_2)] \\ & = \exp[-\rho_0^{-2}(\mathbf{s}_1 - \mathbf{s}_2)^2], \end{aligned} \quad (6)$$

where $D_\psi(\mathbf{s}_1 - \mathbf{s}_2)$ represents the wave structure function, and $\rho_0 = (0.545C_n^2k^2L)^{-3/5}$ is the coherence length of a spherical wave propagating in the turbulent medium, $C_n^2$ being the structure constant. Here, we note that Rytov method is known to be valid in the weak turbulence regime especially when fourth-order moments such as scintillations are considered. Customarily, weak turbulence is associated with Rytov log amplitude variance $0.307C_n^2k^{7/6}L^{11/6}$ being quite smaller than unity. However, this article concerns the second-order moment by utilizing the wave structure function which is approximated by the phase structure function which is known to be valid not only for the case of "weak fluctuations", but for the case of "strong fluctuations" as well, i.e., when $0.307C_n^2k^{7/6}L^{11/6} > 0.5$.

Strictly speaking, given a Kolmogorov spectrum, the wave structure function, $D_\psi(\mathbf{s}_1 - \mathbf{s}_2)$ appearing in Eq. (6), has the following form [13]

$$D_\psi(\mathbf{s}_1 - \mathbf{s}_2) = 2\rho_0^{-2}(\mathbf{s}_1 - \mathbf{s}_2)^{5/3}, \quad (7)$$

which differs from Eq. (6) via the power of the vectorial component. Namely, the power of $(s_1 - s_2)$ in Eq. (6) is 2 (quadratic), whereas the power in Eq. (7) is 5/3 (five thirds). It is well known that five thirds power takes into account not only tip-tilt phase fluctuations but also defocusing and astigmatism, etc., effects. However, approximating five thirds to quadratic will not cause appreciable deviations in terms of the experimentally observed measurements [14,15]. For this reason, below we continue by employing the quadratic approximation of the phase structure function, since this also aids in obtaining simpler and viewable analytical results. Furthermore a better insight into the dependency of the intensity on source and propagation parameters will be gained from an analytic expression that would not be possible, if the integral in Eq. (5) were to be evaluated numerically. We do however test the validity of these statements once more in Section 3. Results and discussions, and within the range of parameters investigated we found out that the exact solution with five thirds power does not yield appreciably different results with the quadratic approximation, except for the cases where high accuracy is required.