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The use of fractional orders in the determination of birefringence of highly dispersive materials by the channelled spectrum method

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Abstract

The channelled spectrum employing polarized light interference is a very convenient method for the study of dispersion of birefringence. However, while using this method, the absolute order of the polarized light interference fringes cannot be determined easily. Approximate methods are therefore used to estimate the order. One of the approximations is that the dispersion of birefringence across neighbouring integer order fringes is negligible. In this paper, we show how this approximation can cause errors. A modification is reported whereby the error in the determination of absolute fringe order can be reduced using fractional orders instead of integer orders. The theoretical background for this method supported with computer simulation is presented. An experimental arrangement implementing these modifications is described. This method uses a Constant Deviation Spectrometer (CDS) and a Soleil Babinet Compensator (SBC).

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Keywords: Birefringence; Polarization; Fractional orders; Soleil Babinet compensator; Constant deviation spectrometer

1. Introduction

Experimental methods for the measurement of refractive index and birefringence of materials have evolved considerably over the years [1–14]. Highly sophisticated instruments are now available commercially for such measurements [15,16]. However, the channelled spectrum (CS) method employing interference of polarized light is still commonly used in laboratories for the study of birefringence and its dispersion [4]. While this method is straightforward and simple for most applications, we have observed certain interesting anomalies while using this method. In this paper, an analysis of these effects and some solutions are presented.

In the CS method, a slice of birefringent specimen is placed between crossed polarizers and illuminated with light of varying wavelengths. For some wavelengths, the phase retardations produced between the ordinary and extraordinary rays is an integral multiple of 2π. The analyzer extinguishes such rays, and correspondingly dark bands are produced in the spectrum of emergent light.

Thus, for neighbouring dark bands at wavelengths λ_0 and λ_1, having absolute fringe orders P and $(P+1)$, respectively, we may write

$$tb_0 = P\lambda_0 \quad (1)$$

$$tb_1 = (P+1)\lambda_1 \quad (2)$$

where t is the sample thickness and b_0 and b_1 are the birefringence values at λ_0 and λ_1.

Here the fringe of order P is treated as an arbitrary “reference fringe”.

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1.1. Estimation of absolute fringe order

The conventional methods employed for estimation of the absolute fringe order are:

1.1.1. Method 1

Using an approximation that the dispersion of birefringence over neighbouring orders is negligible [1], we may write $b_0 = b_1 = b$.

This gives,

$$P = \frac{\lambda_1}{(\lambda_0 - \lambda_1)} \tag{3}$$

We may use pairs of adjacent fringe wavelengths [1] to calculate corresponding successive values of P.

1.1.2. Method 2

Employing the same approximation as in method1, we may calculate P from Eq. (3) for just one longest wavelength pair λ_0 and λ_1 and thereafter assign orders $P+1$, $P+2, \ldots (P+i)$ to neighbouring fringes progressively towards the shorter wavelength side of the spectrum [1].

1.1.3. Method 3

Some authors [1,7] however, extend the no-dispersion assumption further over several orders of fringes to get a "better" estimate of P, as follows:

$$tb_0 = P\lambda_0 \tag{4}$$

for the reference fringe and

$$tb_0 = (p+i)\lambda_i \tag{5}$$

for n successive fringes.

where $i = 1, 2, 3 \ldots n$.

This gives

$$P = \frac{\mathrm{i}\lambda_i}{(\lambda_0 - \lambda_i)} \tag{6}$$

P is then averaged over several values of i and approximated to the nearest integer. Thus, the absolute fringe order for the *reference fringe* in the fringe system is estimated. In order to calculate the dispersion of birefringence, the assumption is then rolled back and the general formula

$$bi = \frac{(P+i)\lambda_i}{(\lambda_0 - \lambda_i)t} \tag{7}$$

is used, with the estimated value of P.

These three methods have their own relative merits and demerits when used in a typical experimental situation. In the CS method, the experimental data are primarily fringe wavelength values and the crystal thickness.

Method 1 uses each pair of wavelengths to calculate fringe order independently. Though this method makes use of several experimental fringe wavelength values, it is sensitive to experimental errors (particularly random errors) in determining these wavelengths. This method is therefore not very effective for the study of dispersion of birefringence.

In method 2, the order calculation is done for just *one* pair of wavelengths as reference, as explained in Section 1.1.2. Thus, any experimental errors occurring in this measurement would get carried forward to the subsequent orders, which are calculated based on this reference.

Method 3 benefits from the law of averages. *All* the data points are used to calculate the *reference order*. This minimizes the effect of random errors in the experimental determination of λ.

However, if the dispersion of birefringence were large, a systematic error would creep in. This is because the closest pair would give one order while the farthest pair would give a significantly different order.

In this paper, these three methods are applied first to a typical set of experimental results. They are then applied to simulated fringe systems and the results are analysed. Particular errors associated with the simplifying assumptions mentioned earlier are discussed. Finally, some experimental work around solutions for minimizing these errors in terms of non-integer fringe orders are presented.

2. Materials and methods

Single crystals of potassium acid phthalate (KAP) were grown in our laboratory by the slow evaporation solution growth technique with triple distilled water as the solvent. Merck AR Grade KAP was used.

The experimental set up that was used to study the birefringence of these materials using the CS method is shown in Fig. 1. A constant deviation spectrometer (CDS) was first calibrated using standard procedures. A 500 W tungsten-halogen lamp was then used to illuminate the slit of the CDS. The CDS has an additional pinhole placed at the focal plane of the eyepiece at the exit side. This provides a collimated beam for studying the samples. A crystal plate (C), was mounted on an upright capable of rotating it in a vertical plane and placed between the polarizer (P) and the analyzer (A). The crystal was oriented with the slow axis at 45° to the axis of polarizer. Finally, the light emerging from the analyzer was examined through the microscope (M). The advantage of using a microscope is that interference can be observed even if the crystal area is as small as 0.004 m^2.

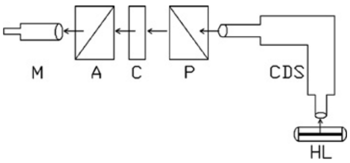

Fig. 1. Experimental setup for the study of dispersion of birefringence M, microscope; A, analyzer; C, crystal sample; P, polarizer; CDS, constant deviation spectrometer and HL, halogen lamp.