Available online at www.sciencedirect.com
ScienceDirect
Optics Communications 274 (2007) 37–42
OPTICS COMMUNICATIONS
www.elsevier.com/locate/optcom

Design of three-dimensional superresolving binary amplitude filters by using the analytical method

Yaoju Zhang *

College of Physics and Electronic Information, Wenzhou University, Wenzhou 325027, China

Received 30 August 2006; received in revised form 27 January 2007; accepted 29 January 2007

Abstract

An analytical method known for phase filters is applied to amplitude filters. The method is based on the use of a series of figures of merit, such as the axial and the transverse gains and the Strehl ratio, which characterize the point-spread-function distribution near the focal region. As a practical implementation, we have applied this method to analytically design the superresolving three-zone amplitude filters and obtained a complete reshaping of the spatial intensity distribution. Regions of the radii of the filters with the transverse, axial or three-dimensional superresolutions are shown analytically. The maximum possible transverse or axial gain is derived for the certain Strehl ratio.

PACS: 42.25.Fx; 42.79.Ci; 42.30.Lr

Keywords: Binary amplitude filter; Figures of merit; Superresolution; Diffraction

1. Introduction

The intensity distribution in the three-dimensional image of a point source provided by an imaging system, i.e., the intensity point-spread-function (PSF) of the system, can be controlled by filters. Superresolving filters have considerable significance in many applications, such as the optical data storage [1,2], the confocal scanning microscopy [3], and laser free-space communications [4]. Superresolution was proposed in 1952 by Toraldo di Francia [5]. Some superresolving filters are based on phase-only [6–10] or on hybrid amplitude-phase profiles [11,12] while others use amplitude-only profiles. In studies of superresolution by discrete-amplitude [13–18] and continuous-amplitude filters [19–21], the binary amplitude filters composed of transparent and opaque zones are more frequently used because of their proper performance and simplicity. Furthermore, it is easier to manufacture them for mass production by using available diffractive optics production methods and replication technologies. Some examples of annular binary amplitude filters are the three-zone filters which are designed to increase the axial [13–16] and transverse resolutions [17], or three-dimensional (3D) resolution [18].

Recently, Canales et al. introduced a fast and simple method for the analytical design of superresolving binary phase-only pupil filters [7]. It is of great interest to design pupil filters whose parameters are analytically derived from the figures of merit desired for the light intensity distribution. In this paper we apply this method of the figures of merit to the analytical design of the superresolving binary amplitude filters.

2. Basic theory for filter design

Let us consider a real pupil function $P(\rho)$ where ρ is the normalized radial coordinate over the pupil plane. For a converging monochromatic spherical wave front passing through the center of the pupil, the normalized 3D

* Tel.: +86 13857795510; fax: +86 577 8837 3109.
E-mail address: yaojuzhang@sohu.com.

0030-4018/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.optcom.2007.01.062

intensity distribution, i.e., PSF, in the focal region can be written as

$$I(v,u) = \left|2\int_0^1 P(\rho)J_0(v\rho)\exp(\mathrm{i}u\rho^2/2)\rho\,\mathrm{d}\rho\right|^2, \tag{1}$$

where v and u are radial and axial dimensionless optical coordinates with origin at the geometrical focus, and J_0 is the Bessel function of the first kind of order zero.

According to the theories of Sheppard and Hegedus [13], the transverse and axial intensity distribution provided by this filter is expanded in a Taylor series near the geometric focus. Within the second-order approximation, $I(v,0)$ and $I(0,u)$ can be expressed as:

$$I(v,0) = I_0^2 - \frac{1}{2}(I_1 I_0)v^2, \tag{2}$$

$$I(0,u) = I_0^2 - \frac{1}{4}(I_2 I_0 - I_1^2)u^2, \tag{3}$$

where I_n is the nth moment of the pupil function, defined as

$$I_n = 2\int_0^1 P(\rho)\rho^{2n+1}\,\mathrm{d}\rho. \tag{4}$$

From Eqs. (2) or (3) an expression for the Strehl ratio can be obtained:

$$S = I_0^2. \tag{5}$$

The transverse and axial gains, which are defined as the ratio between the squared widths of the parabolic approximation of PSF without and with the pupil filter, are given respectively by

$$G_\mathrm{T} = 2I_1/I_0, \tag{6}$$

$$G_\mathrm{A} = 12(I_2 I_0 - I_1^2)/I_0^2. \tag{7}$$

For three-zone binary amplitude filters, the pupil function can be expressed as

$$P(\rho) = \begin{cases} 1, & 0 \leqslant \rho \leqslant a \\ 0, & a < \rho \leqslant b \\ 1, & b < \rho \leqslant 1 \end{cases}, \tag{8}$$

where the radii of three-zones are a, b, and 1, respectively. Substituting Eq. (8) into (4) and then substituting into Eqs. (5)–(7), we obtain

$$S = (a^2 - b^2 + 1)^2, \tag{9}$$

$$G_\mathrm{T} = \frac{1 + a^4 - b^4}{1 + a^2 - b^2}, \tag{10}$$

$$G_\mathrm{A} = \frac{4(1 + a^6 - b^6)(1 + a^2 - b^2) - 3(1 + a^4 - b^4)^2}{(1 + a^2 - b^2)^2}. \tag{11}$$

From these expressions of the figures of merit above, the radii of the suprresolving filters can be obtained by using the analytical method. There are three cases which are analyzed in the following sections. The first case is that a certain transverse gain is desired while maintaining an acceptable Strehl ratio. The design of the filter parameters in such case is shown in Section 3. The second case is that a certain axial gain is desired while maintaining an acceptable Strehl ratio, which is analyzed in Section 4. The third case is that the achievement of the 3D superresolution is studied in Section 5.

3. Filter design from G_T and S

An important case is the derivation of the filter radii from G_T and S because transverse superresolution is of great interest in many different practical applications such as the surface microscopy and the optical storage. In such case, the values of the radii (a,b) can be solved from Eqs. (9) and (10), as

$$a = \left[\frac{\sqrt{S}(2 - G_\mathrm{T}) - S}{2(1 - \sqrt{S})}\right]^{1/2}, \tag{12}$$

$$b = \left[\frac{2 + S - \sqrt{S}(2 + G_\mathrm{T})}{2(1 - \sqrt{S})}\right]^{1/2} \left(= [a^2 + 1 - \sqrt{S}]^{1/2}\right). \tag{13}$$

It is well known that any attempt to superresolve an object will lead to the decrease of the Strehl ratio. A certain level of Strehl ratio is required for the optical recording and the optical microscopy. In optical readout a certain reduction in Strehl ratio is acceptable. For applications of the optical storage the minimum acceptable value of Strehl ratio is generally in the range of 0.4–0.5, whereas for the confocal scanning microscopy this range of the value is generally from 0.2 to 0.3. So, the Strehel ratio must be considered for designing a superresolving filter. To this aim, we express the transverse gain as a function of the Strehl ratio, giving

$$G_\mathrm{T} = 2 - \frac{2a^2(1 - \sqrt{S}) + S}{\sqrt{S}}. \tag{14}$$

If we fix a certain value of Strehl ratio S in Eq. (14) and let

$$\frac{\mathrm{d}G_\mathrm{T}}{\mathrm{d}a} = 0, \tag{15}$$

we can easily find that when $a = 0$ the maximum possible transverse gain is attained as

$$G_\mathrm{T}^\mathrm{m} = 2 - \sqrt{S}. \tag{16}$$

It is noted that $a = 0$ merely means for the two-zone amplitude. So, we can conclude that two-zone amplitude filters are the best option for the transverse superresolution.

The regions of values of the radii a and b which give the transverse superresolution can be obtained from Eqs. (9) and (10), by setting $G_\mathrm{T}(a,b) \geqslant 1$:

$$G_\mathrm{T}(a,b) \geqslant 1 \Longleftrightarrow \begin{cases} a \leqslant \sqrt{\sqrt{S}/2} \\ a \leqslant b \leqslant \sqrt{1 - \sqrt{S}/2} \end{cases}. \tag{17}$$

Fig. 1 shows the region of transverse superresolution, defined by Eq. (17). Fig. 2 shows the transverse gain as a function of the Strehl ratio for different values of radius