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A semi-classical approach to two-frequency solitons in a three-level cascade atomic system

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Abstract

A semi-classical theory of two intense optical fields interacting with a third-order non-linear medium composed of a three-level cascade atomic system is presented. It is predicted that non-linear atom-field interactions allow the formation of two-frequency bright, dark and grey spatial solitons. We demonstrate through numerical simulations and analytic stability analysis that the bright and grey solitons are stable.

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1. Introduction

Single or multi-frequency strong optical fields propagating through a Kerr-like medium exhibit some important non-linear effects such as self- and cross-phase modulation, modulational instability and multi-wave mixing [1]. In the case of self-phase modulation of a single frequency optical wave the medium can become self-focused or defocused depending upon the sign of the third-order susceptibility [2]. In a self-focusing medium a balance between the self-focusing and diffraction of the beam may cause the optical beam to propagate through the medium without changing its transverse profile. These sorts of waves are known as optical bright solitons. Conversely, a defocussing non-linear medium can support the propagation of dark solitons. Solitons have some very important possible applications in optical switching and beam processing applications. Experimentally, bright [3–5] and dark solitons [6,7] have been studied in a variety of non-linear materials.

In the case of multi-frequency optical wave propagation through a Kerr-like medium, the two waves not only interact with the medium but also with each other. Further, the waves induce cross-phase modulation of each other. In Ref. [8,9], such interactions have been considered in detail and they have shown that waves with two different frequencies and identical beam widths can propagate in the form of two-frequency spatial solitons. In the studies presented in [10,11] the collisions between two bright solitons have been considered and interesting results such as splitting, switching and steering of one beam by the other have been predicted.

In this paper we investigate the propagation of two intense optical beams in a three-level atomic system in the cascade configuration. The three-level cascade atomic system has been studied in detail by many groups in quantum optics, non-linear optics and laser physics. In quantum-optics, a number of interesting phenomena such as, phase-sensitive [12] and super radiant amplifications [13], violation of classical effects [14], dipole amplitude square squeezing [15], and phase-dependent fluorescence linewidth narrowing [16], have all been analysed. In the area of non-linear optics, the electromagnetically induced transparency for a probe field in the presence of a strong field [17,18] and field entropy [19] have been considered. In the theory of lasers a number of interesting and potentially important effects have been investigated, including, non-Markovian decay [20], dynamics of a two-photon laser with an injected

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signal [21] and lasing without inversion [22]. Experimentally the three-level cascade atomic system has been studied for polarisation effects in Rubidium [23] and laser cooling and diffusion in metastable helium [24,25].

This paper deals with a Kerr-like medium composed of uniformly distributed three-level atomic systems in the cascade configuration for which the atoms are initially in the ground level (see Fig. 1). Two instense optical fields of different frequencies induce the top to intermediate and intermediate to ground atomic level dipole allowed transitions. The transition from the top to the ground levels are dipole forbidden transitions. We also consider a closed atomic system where the top level decays to the intermediate level and the intermediate level to the ground level with different decaying rates. We calculate the steady-state expressions for the third-order susceptibilities experienced by the two fields while propagating through the medium. We then use these susceptibility expressions to determine the non-linear polarisation induced by the two fields.

In Section 3, we consider the case when two optical fields propagate through a thin slab of medium that contains the three-level cascade atomic system described above. We use the expressions of non-linear polarisation experienced by the two waves to show that the two optical waves of the same width can propagate in the form of spatial solitons. We show that for different conditions of atom-field detuning and wavelengths of the two waves, a pairing of two bright, two dark and a bright-dark combination of beams (a grey soliton) are made possible because of the third-order non-linearity of the medium.

In Section 4 we perform the analytical stability analysis of firstly, the plane waves solutions to our model. This is the well known modulational stability analysis. Secondly, we consider the linear stability analysis of our soliton solutions and examine the behaviour of the maximal eigenvalue.

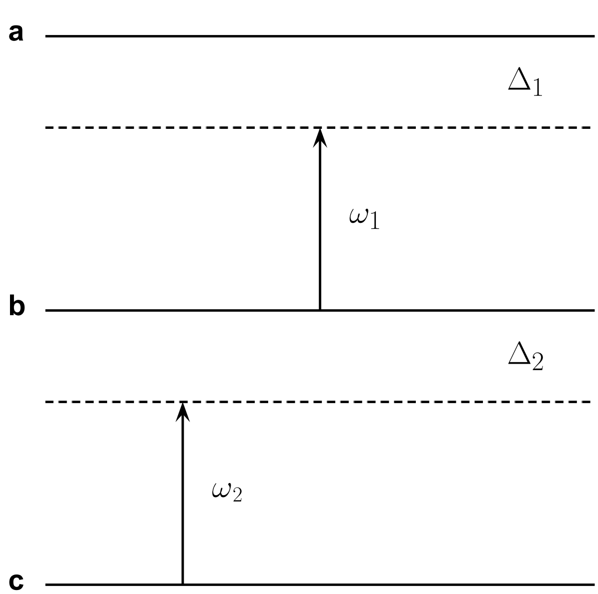

Fig. 1. Three-level atomic system in cascade configuration.

Finally we analyse the stability of these solitons by performing numerical simulations of the governing equations. Our results show that bright and grey solitons can experience stable propagation through the medium. In the case of the dark soliton we consider two situations: (i) when both the fields have almost equal strength; (ii) when one field is much weaker than the other. In both cases the soliton is ultimately unstable, as predicted by the stability analysis of Section 4, but when different strength fields propagate together the stronger field can generate a waveguide for the weaker field thus enabling the latter field to propagate a longer distance before the soliton as a whole becomes unstable.

2. Two-field interaction in a Kerr-law medium

In this section we will consider the interactions of two different frequency fields with a Kerr-law medium that is composed of a closed three-level atomic system in a cascade configuration as shown in Fig. 1.

We also consider that the medium is in gaseous form and that the atoms do not interact with each other. The fields of frequencies ω_1 and ω_2 induce the dipole allowed transitions between levels $|a\rangle$ and $|b\rangle$, and levels $|b\rangle$ and $|c\rangle$, respectively. The levels $|a\rangle$ and $|c\rangle$ have the same parity and do not support dipole allowed transitions. We also consider that the medium is not perturbed externally and that initially most of the atom are at the ground level $|c\rangle$.

The atom-field interaction can be described through Maxwell's equation

$$\nabla \times \nabla \times \vec{E} + \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = -\frac{1}{c^2}\frac{\partial^2 \vec{P}_{NL}}{\partial t^2}, \tag{1}$$

where in the case of two plane polarised fields of frequencies ω_1 and ω_2, we can define the total electric field as

$$E(x,y,z,t) = E_1(x,y,z,t) + E_2(x,y,z,t). \tag{2}$$

For a Kerr-like medium the polarisation can be written as

$$P_{NL}(x,y,z,t) = (\chi^{(1)} + \chi^{(3)}|E(x,y,z,t)|^2)E(x,y,z,t), \tag{3}$$

where $\chi^{(1)}$ and $\chi^{(3)}$ are the first and third-order non-linear susceptibilities. Furthermore the refractive index experienced by the field is of the form

$$n(E) = n_0 + n_2|E|^2, \tag{4}$$

or equivalently as

$$\begin{aligned} n(E) &= (1 + \chi^{(1)} + \chi^{(3)}|E|^2)^{1/2} \\ &\approx n_0 + \frac{\chi^{(3)}}{2n_0}|E|^2. \end{aligned} \tag{5}$$

If we consider that the two fields interact with a homogeneously distributed three-level atomic system in the cascade configuration then, following [1], the polarisation induced by two fields will be the average dipole moment per unit volume and can be defined as

$$\begin{aligned} P(\mathbf{r},t) &= N\mathrm{Tr}(\mu\rho), \\ &= N[\mu_{ac}\rho^{(3)}_{ca} + \mu_{bc}\rho^{(3)}_{cb}] + c.c., \end{aligned}$$