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Single photon self-interference via inelastic two-wave mixing in a coherently prepared cold medium

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Abstract

We investigate a coherently prepared cold medium for efficient single-photon inelastic two-wave mixing (ITWM), maximum Fock state entanglement and single photon self-interference. We show the possibility of generating maximally entangled single-photon state, and near 100% conversion efficiency for generating a frequency shifted TWM photon by proper choice of medium length and concentration. In addition, we demonstrate a new type of transparency effect produced by an efficient *single photon self-interference*, a transparency effect that is very different from the conventional electromagnetically induced transparency (EIT) process.
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Quantum entanglement [1–3] is one of the most striking features of quantum mechanics. The entanglement of the quantum states of separate particles, such as the entanglement of photon pairs [4,5], plays a crucial role in quantum information science [6], and has been intensively studied. The entanglement of multiple Fock states with a single slow or ultra slow photon (or a few photons), however, has only recently been predicted using perfectly efficient, pair-wise four-wave mixing (FWM) technique [7]. Rapid development in the design of sources based on quantum dots coupled to optical cavities has opened the possibility of commercial pulsed sources emitting single photons on-demand with Fourier transform limited bandwidths. Such single photon sources with well defined polarization state and direction of propagation are likely in the near future [8,9].

In this letter, we discuss a perfectly efficient inelastic two-wave mixing (TWM) [10] process for achieving maximum entanglement of single-photon Fock states using a coherently prepared medium. In addition to the prediction of 100% photon conversion efficiency, we show a transparency produced by a *single* pump-photon wave packet due to single photon *self-interference*. This latter effect is very different from the conventional electromagnetically induced transparency (EIT) [11] process where an intense control field is always required. These unique and important results have never been reported and may have important applications in quantum information technology.

We consider a system of life-time broadened four-level atoms that is coherently prepared so that the following long-lived coherent superposition state is obtained:

$$|\Psi(z,t)\rangle = A_1 e^{-i\omega_1 t}|1\rangle + A_3 e^{-i\omega_3 t} e^{i(\omega_3-\omega_1)z/c}|3\rangle, \quad (1)$$

where A_1 and A_3 are the amplitudes of the states $|1\rangle$ and $|3\rangle$. This state vector describes the atomic system after the excitation of the four-level system by two classical fields E_1 and E_2, as depicted in Fig. 1a. Thus, after the laser fields E_1 and E_2 exit from the medium, the atoms will be in the state described by Eq. (1). We note that in forming the coherent superposition state a grating has been written to the medium to alleviate problems associated with phase mismatch when a pump photon at ω_p is introduced at a later time to initiate the inelastic two-wave mixing.

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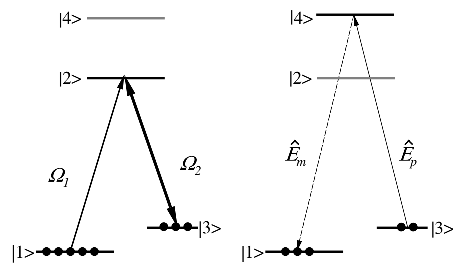

Fig. 1. (a) Left panel: Energy level diagram showing states $|j\rangle$ $(j = 1,2,3)$ are used in preparing the coherent superposition between states $|1\rangle$ and $|3\rangle$. The state $|4\rangle$ (in light color) is used in the time delayed two-wave mixing. (b) Right panel: Energy level diagram showing the states used when the time delayed pump field $(\widehat{E}_p)$ is incident to induce two-wave mixing field $(\widehat{E}_m)$ in a system where a coherent superposition of states (i.e., Eq. (1)) has been established. During this TWM process, the state $|2\rangle$ (in light color) is not needed.

The above described coherent superposition state can be produced with various coherent population transfer techniques routinely used in laboratories. For instance, in the optical frequency domain one may use a pair of collinear, circularly polarized lasers, Ω_j $(j = 1,2)$ with Ω_1 and Ω_2 being the Rabi frequencies of a pulsed (coupling transition $|1\rangle \to |2\rangle$) and a continuous wave (coupling $|3\rangle \to |2\rangle$) fields, respectively (Fig. 1a). Under this circumstances, a coherent mixture of the ground state $|1\rangle$ and the lowest excited state $|3\rangle$ can be created, yielding $A_1 = \Omega_2/\Omega$, $A_3 = -\Omega_1/\Omega$ where $\Omega = \sqrt{\Omega_1^2 + \Omega_2^2}$ [12]. The coherence can be stored by rapidly switching off Ω_2. Alternatively, in the radio-frequency domain one may use microwaves to achieve the direct non-electric dipole coupling of states $|1\rangle$ and $|3\rangle$.

After the coherence is prepared or stored, state $|2\rangle$ will not be needed for the subsequent TWM process. Thus, a three-state system (i.e., $|1\rangle$, $|3\rangle$, and $|4\rangle$, see Fig. 1b) is sufficient for our discussion of TWM. Therefore, the start point of the present study is such a coherently prepared system with all atoms being in the prepared state in Eq. (1) with A_1 and A_3 given above.

To generate single TWM photon we inject a single pump-photon wave packet into the prepared system as described above. If the delay between the time when the pump-photon wave packet is introduced and the time when the coherent preparation is completed is small enough that the decay of the coherence is negligible, we effectively have a three-state system as shown in Fig. 1b. We emphasize that this is a three-state lambda system, yet the conventional EIT process plays *no* role. To make this point clear, we stress three differences between the three-state lambda system shown in Fig. 1b and the three-state lambda system typically used in the conventional EIT scheme. First, in the conventional EIT scheme, the control field couples two *empty* states and it must be sufficiently intense to drive the upper state of a one-photon transition transparent. In our case the single photon pump field couples two states, one of them having substantial population. Such a weak field can never generate a transparency in the upper state of a one-photon transition. Secondly, in the conventional EIT scheme, the probe and control fields are both *externally* supplied fields whereas in our case one of the participating fields is an *internally* generated single photon field. Furthermore, we will describe a transparency produced by the *single* pump photon as a result of *self-interference*. This is a remarkable effect because the mechanism of producing the transparency is very different from that of conventional EIT scheme. Indeed, such an induced transparency does not exist in the conventional EIT scheme as it depends critically on the internally generated field.

We use the Heisenberg representation to calculate the atomic dynamics and the electromagnetic fields, but we treat the elements of the density matrix $\hat{\rho}_{13}$, $\hat{\rho}_{33}$, and $\hat{\rho}_{11}$ as *c-numbers* whose values are not changed by the passage of a single-photon wave packet through the medium. This is consistent with the assumption of a long-lived coherent superposition state.

The first step of the derivation involves the calculation of the polarization operators that to be used to construct the source terms, in the form of operators, which will be needed in the operator form of Maxwell equations for the single photon pump (E_p) and TWM (E_m) fields. One thus derive the operator equations of motion for density operators ρ_{14} and ρ_{34}. If we write

$$\hat{\rho}_{14} = \widehat{S}_m e^{-i\omega_m t} e^{i\omega_m z/c},$$
$$\hat{\rho}_{34} = \widehat{S}_{p'} e^{-i\omega_{p'} t} e^{i\omega_{p'} z/c},$$

then instead of the operator equations of motion for ρ_{14} and ρ_{34} we have the following two equations of motion for the operators $\widehat{S}_p$ and $\widehat{S}_m$:

$$\frac{\partial \widehat{S}_m}{\partial t} = i d_4 \widehat{S}_m + i\frac{|\Omega_2|^2}{|\Omega|^2}\widehat{W}_m - i\frac{\Omega_2^*\Omega_1}{|\Omega|^2}\widehat{W}_p, \tag{2a}$$

$$\frac{\partial \widehat{S}_p}{\partial t} = i d_4 \widehat{S}_p + i\frac{|\Omega_1|^2}{|\Omega|^2}\widehat{W}_p - i\frac{\Omega_1^*\Omega_2}{|\Omega|^2}\widehat{W}_m, \tag{2b}$$

where $d_4 = \delta_4 + i\gamma_4/2$ and

$$\widehat{W}_m = D_{41}\widehat{E}_m/\hbar, \quad \widehat{W}_p = D_{43}\widehat{E}_p/\hbar.$$

Using Eqs. (2a) and (2b), taking the plane-wave and slowly-varying-phase-and-amplitude approximations we arrive at the following Maxwell's equations for the positive frequency part of the complex amplitudes of the pump and TWM field operators:

$$\left(\frac{\partial \widehat{E}_p^{(+)}}{\partial z}\right)_t + \frac{1}{c}\left(\frac{\partial \widehat{E}_p^{(+)}}{\partial t}\right)_z = \frac{i\kappa_{34}\hbar}{D_{43}}\widehat{S}_p, \tag{3a}$$

$$\left(\frac{\partial \widehat{E}_m^{(+)}}{\partial z}\right)_t + \frac{1}{c}\left(\frac{\partial \widehat{E}_m^{(+)}}{\partial t}\right)_z = \frac{i\kappa_{14}\hbar}{D_{41}}\widehat{S}_m, \tag{3b}$$

where $\widehat{S}_p^{(+)}$ and $\widehat{S}_m^{(+)}$ are the operators proportional to the positive frequency part of the polarization amplitudes at the pump and TWM frequencies, respectively, obtainable from the equations of motion for these operators given