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Spectral behavior of partially coherent beam passing through an aperture

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Abstract

Based on the Rayleigh diffraction integral and the expansion of the hard-aperture function into a finite series of complex Gaussian functions, an approximate analytical spectrum expression for a Gaussian schell-model (GSM) beam passing through an aperture is given. The spectral shifts and spectral switches of near-zone off-axis are studied. The effect of the coherence and hard-edge on the spectral behavior are also presented.

Keywords: GSM beam; Spectral shift; Spectral switch

1. Introduction

It has been shown that the spectrum of light emitted from partially coherent source will, in general, change on propagation [1–12], which was verified experimentally in acoustics [4] and in optical experiments [5]. When the source satisfies the scaling law, the spectrum of light remains invariant on propagation [1]. Otherwise, the spectrum will change on propagation, which is due to the correlation-induced spectral changes [1,6]. It is also noted that when a field satisfying the scaling law incident upon an aperture, the far-zone spectrum of the field is different from that in the aperture [7], which is due to the diffraction-induced spectral changes. Further theoretical studies indicate that the spectrum of partially coherent light incident upon an aperture will split into two same height peaks at critical position, a rapid transition of the spectral shift from red to blue, i.e. spectral switches occur [8,9,12]. Spectral switches were investigated in the near-zone on-axis [8] and far-zone [9] and the predictions have been verified experimentally [10,11]. Numerical techniques are required when studying the near-zone spectral behavior [8]. In this paper, we investigate the near-zone off-axis spectrum of GSM beam passing through an aperture. In Section 2 an approximate analytical spectrum expression for a GSM beam passing through an aperture is derived by expanding the hard-aperture function into a finite terms of complex Gaussian functions. In Section 3 the spectral shifts and spectral switches of near-zone off-axis is studied in detail. The effect of the coherence and hard-edge on the spectral behavior are also presented.

2. Spectrum of Gaussian schell-model beam through an aperture

Consider a beam incident upon an aperture with radius a. For the simplicity and without the loss of generality, we only consider two-dimensional GSM beam. Therefore, the cross-spectral density function of the incident GSM beam at the $z=0$ plane can be expressed as [12,13].

$$W(x_1',x_2',0;\omega)=S^{(0)}(\omega)\exp\left[-\frac{x_1'^2+x_2'^2}{w_0^2}\right]\exp\left[-\frac{(x_1'-x_2')^2}{2\sigma_\mu^2}\right], \tag{1}$$

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where w_0 and σ_μ are the waist width and the root-mean-square (rms) spatial correlation distance, respectively. The corresponding hard-edged function can be written as

$$H(x) = \begin{cases} 1, & |x| \leqslant a \\ 0, & |x| > a \end{cases}. \tag{2}$$

As known, the hard-edged function can be expressed as a finite terms of complex Gaussian functions [14]

$$H(x) = \sum_{n=1}^{l} A_n \exp\left[-\frac{B_n x^2}{a^2}\right]. \tag{3}$$

Here a is the half width of the aperture, and the expansion and Gaussian coefficients A_n and B_n are evaluated by a computer optimization, listed in Table 1 of Appendix. Connecting Eqs. (2) and (3), the propagation of the cross-spectral density function through a hard-edged aperture obeys the generalized Rayleigh diffraction integral [13,15]

$$\begin{aligned} S(x,z;\omega) &= W(x,x,z;\omega) \\ &= \frac{1}{\lambda}\int_{-a}^{a}\int_{-a}^{a} W(x_1',x_2',0;\omega)\frac{\cos\theta_1\cos\theta_2}{R_1R_2} \\ &\quad \times \exp[ik(R_2-R_1)]dx_1'dx_2' \\ &= \frac{1}{\lambda}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W(x_1',x_2',0;\omega)H(x_1')H^*(x_2') \\ &\quad \times \frac{\cos\theta_1\cos\theta_2}{R_1R_2}\exp[ik(R_2-R_1)]dx_1'dx_2'. \end{aligned} \tag{4}$$

where $R_j = [(x_j - x_1')^2 + z^2]^{1/2}$, $\cos\theta_j = z/R_j$ $(j = 1, 2)$ and k is wave number. R_j can be approximately expanded into

$$R_j \approx r_j + \frac{x_j'^2 - 2x_j x_j'}{2r_j}, \tag{5}$$

where $r_j = (x_j^2 + z^2)^{1/2}$. After some algebra, we obtain

$$S(x,z;\omega) = \frac{S^{(0)}(\omega)z^2 Z_0}{r^4}\frac{\omega}{\omega_0}\sum_{n=1}^{N}\sum_{m=1}^{N}\frac{A_n A_m^* \eta^2}{\sqrt{1+4\eta^4 l_1 l_2}} \times \exp\left[-\frac{Z_0^2\eta^4 x^2\left(l_1 - \frac{1}{2\eta^2}\right)^2}{w_0^2 r^2 l_1(1+4\eta^4 l_1 l_2)}\left(\frac{\omega}{\omega_0}\right)^2\right], \tag{6}$$

where

$$Z_0 = k_0 w_0^2; \tag{7a}$$

$$\eta = \frac{\sigma_\mu}{w_0} \quad \text{(global coherence parameter)}; \tag{7b}$$

$$\delta = \frac{a}{w_0} \quad \text{(truncation parameter)}; \tag{7c}$$

$$l_1 = 1 + \frac{1}{2\eta^2} + \frac{iZ_0}{2r}\frac{\omega}{\omega_0} + \frac{B_n}{\delta^2}; \tag{7d}$$

$$l_2 = 1 + \frac{1}{2\eta^2} - \frac{iZ_0}{2r}\frac{\omega}{\omega_0} + \frac{B_m^*}{\delta^2}. \tag{7e}$$

Assume that the spectrum of the light $S^{(0)}(\omega)$ is a Gaussian distribution of center frequency ω_0 and bandwidth σ_0, viz.,

$$S^{(0)}(\omega) = S_0 \exp\left[-\frac{(\omega-\omega_0)^2}{2\sigma_0^2}\right], \tag{8}$$

where S_0 is a positive constant. Eq. (6) indicates that the spectrum at z plane depends on three parameters, i.e. the position (x, z), the global coherence parameter η and the truncation parameter δ. Therefore, the spectrum of the field in the near-zone off-axis of the aperture is different from the spectrum in the aperture and will change on propagation.

3. Near-zone off-axis spectral shifts and spectral switches

Numerical calculations are performed by using Eq. (6), typical results are compiled in Figs. 1–4. Fig. 1 gives the normalized spectrum $S(\omega) = \frac{S(x,z;\omega)}{S_{\max}(x,z;\omega)}$ of a GSM beam in the near-zone. The used parameters are $\omega_0 = 3\times10^{15}\ \mathrm{s}^{-1}$, $\sigma_0 = 0.06\times10^{15}\ \mathrm{s}^{-1}$, $w_0 = 1$ mm, $\delta = 0.2$, $\eta = 1$ and $z = Z_0/4$, (a) $x = 3.970$ mm, (b) $x = 3.974$ mm, (c) $x = 3.980$ mm. The relative spectral shift is defined as $\delta\omega/\omega_0 = (\omega_{\max} - \omega_0)/\omega_0$, where $\omega_{\max}$ denotes the frequency at which the spectrum takes its maximum [8]. As shown in Fig. 1a, $\omega_{\max} < \omega_0$. That is to say, the spectrum is red-shifted at $x = 3.970$ mm. With the increasing of x, the second peak increases gradually. When reaches its critical value $x = 3.974$ mm, the second peak has the same height with the first peak, and a rapid transition of the spectral shift from red to blue occurs, as shown in Fig. 1b. This effect is referred as the spectral switch [8]. With further increasing of x, the second peak is larger than the first peak, and $\omega_{\max} > \omega_0$. In the case of $x = 3.980$ mm, the spectrum is blue shifted, as indicated in Fig. 1c.

The relative spectral shifts versus transversal position x with different truncation parameters δ is plotted in Fig. 2, the other parameters are the same in Fig. 1. For the case of $\delta = 0.2$, the relative on-axis spectral shift $\delta\omega/\omega_0 = 0.00119$. As x increases, the $\delta\omega/\omega$ decreases slowly and becomes negative, i.e. red shift occurs. As x continues to increase, the red shift becomes large till reaching its maximum value. At the critical value $x \approx 1.3$ mm, a rapid transition of spectral shift from red to blue occurs, i.e. the first spectral switch occurs. Furthermore, the spectral shift is symmetrical at this position. Other spectral switches occur at $x \approx 2.65$ mm, 2.97 mm, respectively. For the case of $\delta = 0.4$, there are more spectral switches in the region $0 < x < 5$. However, the maximum relative spectral shift becomes smaller at each critical point where spectral switches occur.

Some normalized near-zone off-axis spectrum of GSM beam at $x = 3.974$ mm with different global coherence parameters η are shown in Fig. 3. The other parameters are the same in Fig. 1. The two limiting cases of a fully coherent and an incoherent beam correspond to $\eta \to \infty$ and $\eta \to 0$, respectively. The normalized spectrum is red shifted when $\eta = 0.1$ and blue shifted when $\eta = 10$. As can be seen from Fig. 3, the normalized near-zone off-axis