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Off-axis Gaussian Schell-model beam and partially coherent laser array beam in a turbulent atmosphere

Yangjian Cai [a,b,*], Qiang Lin [b], Yahya Baykal [c], Halil T. Eyyuboğlu [c]

[a] Max-Planck-Research-Group, Institute of Optics, Information and Photonics, University of Erlangen, Staudtstr. 7/B2, D-91058 Erlangen, Germany
[b] Institute of Optics, Department of Physics, Zhejiang University, Hangzhou 310027, China
[c] Department of Electronic and Communication Engineering, Çankaya University, Öğretmenler Cad. 14 Yüzüncüyıl, 06530 Balgat, Ankara, Turkey

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Abstract

The propagation of an off-axis Gaussian Schell-model (GSM) beam in a turbulent atmosphere is investigated based on the extended Huygens–Fresnel integral formula. Analytical formulae for the cross-spectral density and corresponding partially coherent complex curvature tensor of an off-axis GSM beam propagating in a turbulent atmosphere are derived. Based on these formulae, the propagation properties of such kind of beam in a turbulent atmosphere are investigated in detail. Furthermore, the methods are extended to investigate the propagation properties of a partially coherent laser array beam in a turbulent atmosphere. The properties of an off-axis GSM beam and a partially coherent laser array beam in a turbulent atmosphere are closely related with the beam parameters and the structure constant of the turbulent atmosphere.

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1. Introduction

Investigation of the propagation of laser beams in a turbulent atmosphere becomes more and more important because of their wide applications in free-space optical communication, remote sensing applications. In the past decades, propagation properties of coherent and partially coherent beams in a turbulent atmosphere have been widely studied both theoretically and experimentally [1–16]. Recently, Ricklin et al. analyzed the application of Gaussian Schell-model (GSM) beam in free-space laser communication, and found that application of partially coherent beam can result in a significant reduction in the bit error rate of the optical communication links compared to coherent beam [17,18]. Shirai et al. studied the directionality and spreading of GSM beam propagating through atmospheric turbulence [19,20]. Dogariu and Amarande studied experimentally the turbulence-induced degradation of partially coherent beams [21]. Korotkova et al. investigated the polarization properties of partially coherent beams in the turbulent atmosphere [22,23]. Roychowdhury et al. examined the change in the polarization of partially coherent electromagnetic beams propagating through the turbulent atmosphere [24]. Ji et al. studied the spectrum properties of a partially coherent beam in a turbulent atmosphere [25]. More recently, Cai and He studied the propagation of a twisted anisotropic GSM beam in a turbulent atmosphere [26]. Lu et al. investigated the degree of coherence of partially coherent electromagnetic beams in a turbulent atmosphere [27]. Up to now, propagation of an off-axis partially coherent beam in a turbulent atmosphere has not been studied.

* Corresponding author. Address: Institute of Optics, Department of Physics, Zhejiang University, Hangzhou 310027, China. Tel.: +86 57187953231.
E-mail address: yangjian_cai@yahoo.com.cn (Y. Cai).

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After passing through an inhomogeneous medium/structure or off-setting complex optical systems, a laser beam may become off-axis (or decentered) [28]. Actually most laser beams are off-axis more or less in practical cases. Therefore, it is necessary to study the propagation properties of a decentered laser beam. In 1973, Casperson put forward the idea of off-axis Gaussian beam [28]. Since then, the off-axis laser beams have been studied extensively [29–37]. More recently, Zheng studied the fractional Fourier transform for an off-axis Gaussian Schell-model beam [38]. In this paper, we study the propagation of an off-axis GSM beam in a turbulent atmosphere by using the tensor method, which is a convenient method for treating the propagation of coherent and partially coherent laser beam [14,15,39]. Analytical formulae for the cross-spectral density and partially coherent complex curvature tensor of an off-axis GSM beam propagating in a turbulent atmosphere are derived, and some numerical examples are given. As an application example, the propagation properties of a partially coherent laser array beam in a turbulent atmosphere are studied.

2. Off-axis GSM beam

In this section, we outline briefly the description of an off-axis GSM beam and its paraxial propagation in free space. GSM beam is a typical partially coherent beam, which can be generated with synthetic acousto-optic holograms [40]. GSM beam is characterized by the cross-spectral density, which can be expressed at $z = 0$ as follows [40,41]:

$$W_0(\boldsymbol{r}_1, \boldsymbol{r}_2, 0) = \exp\left[-\frac{\boldsymbol{r}_1^2 + \boldsymbol{r}_2^2}{4\sigma_{I0}^2} - \frac{(\boldsymbol{r}_1 - \boldsymbol{r}_2)^2}{2\sigma_{g0}^2}\right], \tag{1}$$

where $\boldsymbol{r}_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\boldsymbol{r}_2 = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ are the transverse position vectors at $z = 0$, σ_{I0} and σ_{g0} are the transverse spot width and transverse coherence width of a GSM beam. The cross-spectral density of an off-axis GSM beam at $z = 0$ is expressed as [38]

$$W_0(\boldsymbol{r}_1, \boldsymbol{r}_2, 0) = \exp\left[-\frac{(\boldsymbol{r}_1 - \boldsymbol{r}_0)^2 + (\boldsymbol{r}_2 - \boldsymbol{r}_0)^2}{4\sigma_{I0}^2} - \frac{(\boldsymbol{r}_1 - \boldsymbol{r}_2)^2}{2\sigma_{g0}^2}\right], \tag{2}$$

where $\boldsymbol{r}_0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$ is called the off-axis parameter, the irradiance distribution is given as $I(\boldsymbol{r}, 0) = W_0(\boldsymbol{r}, \mathbf{r}, 0) = \exp[-(\boldsymbol{r} - \boldsymbol{r}_0)^2 / 2\sigma_{I0}^2]$ and the beam center of the beam spot is located at $(x_0 \quad y_0)$. After some arrangement, Eq. (2) can be expressed in following tensor form [38]

$$W_0(\tilde{\boldsymbol{r}}, 0) = \exp\left[-\frac{\mathrm{i}k}{2}(\tilde{\boldsymbol{r}} - \tilde{\boldsymbol{r}}_0)^{\mathrm{T}} \boldsymbol{M}_1^{-1} (\tilde{\boldsymbol{r}} - \tilde{\boldsymbol{r}}_0)\right], \tag{3}$$

where $\tilde{\boldsymbol{r}}^{\mathrm{T}} = \begin{pmatrix} \boldsymbol{r}_1^{\mathrm{T}} & \boldsymbol{r}_2^{\mathrm{T}} \end{pmatrix} = (x_1 \quad y_1 \quad x_2 \quad y_2)$, $\tilde{\boldsymbol{r}}_0^{\mathrm{T}} = (x_0 \quad y_0 \quad x_0 \quad y_0)$, $k = 2\pi/\lambda$ is the wave number, λ is the wavelength, $\boldsymbol{M}_1^{-1}$ is the partially coherent complex curvature tensor given by

$$\boldsymbol{M}_1^{-1} = \begin{pmatrix} \left(-\frac{\mathrm{i}}{2k\sigma_{I0}^2} - \frac{\mathrm{i}}{k\sigma_{g0}^2}\right)\boldsymbol{I} & \frac{\mathrm{i}}{k\sigma_{g0}^2}\boldsymbol{I} \\ \frac{\mathrm{i}}{k\sigma_{g0}^2}\boldsymbol{I} & \left(-\frac{\mathrm{i}}{2k\sigma_{I0}^2} - \frac{\mathrm{i}}{k\sigma_{g0}^2}\right)\boldsymbol{I} \end{pmatrix}, \tag{4}$$

where $\boldsymbol{I}$ is a 2×2unit matrix.

Substituting Eq. (3) into the generalized Huygens–Fresnel integral formula for treating the propagation of a partially coherent beam in free space [39], we obtain the following analytical propagation formula for the cross-spectral density of an off-axis GSM beam in free space

$$W(\tilde{\boldsymbol{\rho}}, z) = \left[\det(\widetilde{\boldsymbol{I}} + \widetilde{\boldsymbol{B}}\boldsymbol{M}_1^{-1})\right]^{-\frac{1}{2}} \exp\left[-\frac{\mathrm{i}k}{2}\tilde{\boldsymbol{\rho}}^{\mathrm{T}}\boldsymbol{M}_2^{-1}\tilde{\boldsymbol{\rho}}\right] \exp\left[-\frac{\mathrm{i}k}{2}\tilde{\boldsymbol{r}}_0^{\mathrm{T}}\boldsymbol{M}_2^{-1}\tilde{\boldsymbol{r}}_0\right] \exp\left[\mathrm{i}k\tilde{\boldsymbol{r}}_0^{\mathrm{T}}\boldsymbol{M}_2^{-1}\tilde{\boldsymbol{\rho}}\right], \tag{5}$$

where $\widetilde{\boldsymbol{I}}$ is a 4×4 unit matrix, $\tilde{\boldsymbol{\rho}}^{\mathrm{T}} = \begin{pmatrix} \boldsymbol{\rho}_1^{\mathrm{T}} & \boldsymbol{\rho}_2^{\mathrm{T}} \end{pmatrix} = \begin{pmatrix} \rho_{1x} & \rho_{1y} & \rho_{2x} & \rho_{2y} \end{pmatrix}$, $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are the transverse position vectors on the output plane. $\boldsymbol{M}_1^{-1}$ and $\boldsymbol{M}_2^{-1}$ denote the partially coherent complex curvature tensors on the input and output planes, respectively. They satisfy the following formula:

$$\boldsymbol{M}_2^{-1} = (\boldsymbol{M}_1 + \widetilde{\boldsymbol{B}})^{-1}, \tag{6}$$

where