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Higher-order squeezing and photon statistics in fourth harmonic generation

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Abstract

Squeezing of the electromagnetic field is investigated in fundamental mode in fourth harmonic generation under a short time approximation based on quantum mechanical approach. The occurrence of squeezing in field amplitude and in higher power amplitude (up to fourth-order) in fundamental mode has been studied. It is found to be dependent on phase of field amplitude of the fundamental mode. The dependence of squeezing on the photon number has also been investigated. It is found that squeezing increases non-linearly. It has also been found that the photon statistics of the field in fundamental mode is sub-Poissonian. The signal to noise ratio has been studied in different order. It is found that signal to noise ratio is higher in lower order.

Keywords: Harmonic generation; Non-linear optics; Photon number; Squeezing; Sub-Poissonian

1. Introduction

Squeezed states are the minimum uncertainty states with reduced noise below the vacuum limit in one of the canonical conjugate quadrature of the field amplitude. Such states are the closest counterpart to a coherent state. The coherent states do not exhibit non-classical effects but a superposition of coherent states can exhibit various non-classical effects, such as, squeezing (lower as well as higher) and sub-Poissonian photon statistics (antibunching). Superposition of coherent states can be generated in interaction of coherent states with non-linear medium. It has been found that some coherent states exhibit squeezing but not sub-Poissonian statistics [1] and some coherent states exhibit sub-Poissonian statistics but not squeezing, e.g. a pure number state exhibits antibunching but not squeezing because of the complete lack of phase information. In some processes superposition of two coherent states of identical mean photon number but different phases can exhibit both squeezing and antibunching [2,3].

With the development of techniques of higher-order correlation measurements, Hong and Mandel [4] and Hillery [5] introduced the notion of higher-order squeezing. Zhan [6] proposed the generation of amplitude cubed squeezing in the fundamental mode in second and third harmonic generation. Squeezed states have several possible applications such as gravitational wave detection [7–10], optical storage [11], teleportation [12], high precision measurements [13], and so on. It has been shown that such states can be generated in various non-linear optical processes such as, harmonic generation [14–17], multi-wave mixing [18–22], Raman processes [23] and Hyper-Raman processes [24] etc.

The results of this paper show the simultaneous squeezing and sub-Poissonian effects when both are maximum. Fourth harmonic generation in fundamental mode is one of such distinguished examples where light exhibits both squeezing and sub-Poissonian photon statistics at the same time as presented in this paper.

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2. Definition of squeezing and higher-order squeezing

Squeezed states of an electromagnetic field are the states with reduced noise below the vacuum limit in one of the canonical conjugate quadrature. Normal squeezing is defined in terms of the operators

$$X_1 = \frac{1}{2}(A + A^\dagger) \quad \text{and} \quad X_2 = \frac{1}{2i}(A - A^\dagger) \tag{1}$$

where X_1 and X_2 are the real and imaginary parts of the field amplitude, respectively. A and $A^\dagger$ are slowly varying operators defined by

$$A = a\mathrm{e}^{\mathrm{i}\omega t} \quad \text{and} \quad A^\dagger = a^\dagger \mathrm{e}^{-\mathrm{i}\omega t} \tag{2}$$

The operators X_1 and X_2 obey the commutation relation

$$[X_1, X_2] = \frac{i}{2} \tag{3}$$

which leads to uncertainty relation ($\hbar = 1$)

$$\Delta X_1 \Delta X_2 \geqslant \frac{1}{4} \tag{4}$$

A quantum state is squeezed in X_i variable if

$$\Delta X_i < \frac{1}{2} \quad \text{for } i = 1 \text{ or } 2 \tag{5}$$

Amplitude-squared squeezing is defined in terms of operators Y_1 and Y_2 as

$$Y_1 = \frac{1}{2}[A^2 + A^{\dagger 2}] \quad \text{and} \quad Y_2 = \frac{1}{2i}[A^2 - A^{\dagger 2}] \tag{6}$$

The operators Y_1 and Y_2 obey the commutation relation $[Y_1, Y_2] = i(2N + 1)$ which leads to the uncertainty relation

$$\Delta Y_1 \Delta Y_2 \geqslant \left\langle \left(N + \frac{1}{2} \right) \right\rangle \tag{7}$$

where N is the usual number operator.

Amplitude-squared squeezing is said to exist in Y_i variable if

$$(\Delta Y_i)^2 < \left\langle \left(N + \frac{1}{2} \right) \right\rangle \quad \text{for } i = 1 \text{ or } 2 \tag{8}$$

Amplitude-cubed squeezing is defined in terms of operators

$$Z_1 = \frac{1}{2}(A^3 + A^{\dagger 3}) \quad \text{and} \quad Z_2 = \frac{1}{2i}(A^3 - A^{\dagger 3}) \tag{9}$$

The operators Z_1 and Z_2 obey the commutation relation

$$[Z_1, Z_2] = \frac{i}{2}(9N^2 + 9N + 6) \tag{10}$$

Relation (10) leads to the uncertainty relation

$$\Delta Z_1 \Delta Z_2 \geqslant \frac{1}{4}(9N^2 + 9N + 6) \tag{11}$$

Amplitude-cubed squeezing exists when

$$(\Delta Z_i)^2 < \frac{1}{4}\langle (9N^2 + 9N + 6) \rangle \quad \text{for } i = 1 \text{ or } 2 \tag{12}$$

Real and imaginary parts of fourth-order amplitude are given as

$$F_1 = \frac{1}{2}(A^4 + A^{\dagger 4}) \quad \text{and} \quad F_2 = \frac{1}{2i}(A^4 - A^{\dagger 4}) \tag{13}$$

The operators F_1 and F_2 obey the commutation relation

$$[F_1, F_2] = \frac{i}{2}(16N^3 + 24N^2 + 56N + 24) \tag{14}$$

and satisfy the uncertainty relation ($\hbar = 1$)

$$\Delta F_1 \Delta F_2 \geqslant \frac{1}{4}\langle (16N^3 + 24N^2 + 56N + 24) \rangle \tag{15}$$

Fourth-order squeezing exists when

$$(\Delta F_i)^2 < \frac{1}{4}\langle (16N^3 + 24N^2 + 56N + 24) \rangle \quad \text{for } i = 1 \text{ or } 2 \tag{16}$$

3. Squeezing of fundamental mode in fourth harmonic generation

Fourth harmonic generation model has been adopted from the works of Chen et al. [25] and is shown in Fig. 1. In this model, the interaction is looked upon as a process which involves absorption of four photons, each having frequency ω_1 going from state $|1\rangle$ to state $|2\rangle$ and emission of one photon of frequency ω_2, where $\omega_2 = 4\omega_1$.

The Hamiltonian for this process is given as follows ($\hbar = 1$)

$$H = \omega_1 a^\dagger a + \omega_2 b^\dagger b + g(a^4 b^\dagger + a^{\dagger 4} b) \tag{17}$$

in the above equation g is a coupling constant for fourth harmonic generation. $A = a\,\exp(\mathrm{i}\omega_1 t)$ and $B = b\,\exp(\mathrm{i}\omega_2 t)$ are the slowly varying operators at frequencies ω_1 and ω_2, respectively, $a(a^\dagger)$ and $b(b^\dagger)$ are the usual annihilation (creation) operators, respectively.

The Heisenberg equation of motion for fundamental mode A is given as ($\hbar = 1$)

$$\frac{\partial A}{\partial t} = \frac{\mathrm{d}A}{\mathrm{d}t} + i[H, A] \tag{18}$$

Using Eq. (17) in Eq. (18), we obtain

$$\dot{A} = -4igA^{\dagger 3}B \tag{19}$$

Similarly,

$$\dot{B} = -igA^4 \tag{20}$$

In this process, we assume the interaction time 't' to be very small. We expand $A(t)$ by using Taylor's series expansion and retaining the terms up to g^2t^2 as

$$A(t) = A - 4igtA^{\dagger 3}B + 2g^2t^2[(12A^{\dagger 2}A^3 + 36A^\dagger A^2 + 24A)B^\dagger B - A^{\dagger 3}A^4] \tag{21}$$

For squeezing of field amplitude in fundamental mode A, the real quadrature component is

$$X_{1A} = \frac{1}{2}[A(t) + A^\dagger(t)] \tag{22}$$