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Polarized optical filtering from general linearly twisted structures

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Abstract

It is shown that a stack of linearly twisted birefringent plates with arbitrary successive twist angle acts as a spectral filter assuming the total twist equals an odd integer of $\pi/2$, and each plate acts as a full wave plate. The classical fan Solc filter is shown to be a very special case of the present concept. Based on this finding, spontaneously twisted structures *analogous* to the twisted liquid crystals and helical sculptured thin films can basically act as polarization filters if their molecular or nano-rod thickness and the local birefringence satisfy the full waveplate condition. Giving a certain number of full wave plates it is shown that there are many ways to arrange their successive twist and get the same transfer function.

Keywords: Optical filters; Anisotropic optical materials; Liquid-crystal devices; Filters; Interference

Polarization interference filters made of a stack of birefringent plates are well known since the first half of the 19th century such as the Solc and the Lyot-Ohmann filters [1–3]. Due to the possibility of electrically tuning these filters when electrooptic crystal or liquid crystal is used, they have attracted wide interest recently for wide variety of applications such as displays, optical telecommunications and hyperspectral imaging where new modified designs have emerged [4–14]. The recent developments in thin solid birefringent and helical films so called chiral sculptured thin films added another important dimension to the subject [15–17]. One of their versions, the so-called fan Solc, consists of a twisted arrangement of birefringent plates such that if the first plate azimuth is σ, the jth plate azimuth is $(2j-1)\sigma$ (consider Fig. 1 with $c = 2$ and $\theta = \pi/2$). When the plates act as full wave plates and $\sigma = \pi/4N$, with N being the number of plates, the structure acts as a band pass filter between parallel polarizers. Although this structure is a special linearly twisted case, it is not the linear twist that one expects to occur spontaneously, in which the azimuth of the jth plate is expected to be $j\sigma$ such as with chiral smectic and cholesteric liquid crystals. In a recent publication Hodgkinson et al. [18] have considered an interesting arrangement of quarter wave thick twisted plates and showed that they exhibit different Bragg reflections for different circular polarization states. In this article I am showing that linearly twisted structures with a general linear succession of azimuths lead to polarized filter action under certain conditions. In this sense the proposed concept can have applications and wide interest in a variety of research and technology areas from living systems to nano-tubes and nano-wires, to liquid crystalline structures and sculptured thin films where birefringent and twisted structures appear.

The geometry of the problem is shown in Fig. 1 where for generality we consider birefringent layers with slow and fast axis having the indices n_s and n_f. The first birefringent layer slow axis is oriented by an azimuth angle σ with respect to the entrance polarizer axis. Each other layer $j+1$ is oriented by an angle $c\sigma$ with respect to the preceding layer j where c is a real number and $j = 1, 2, 3, \ldots, N$, is the serial number of the layer in the stack. The azimuth of

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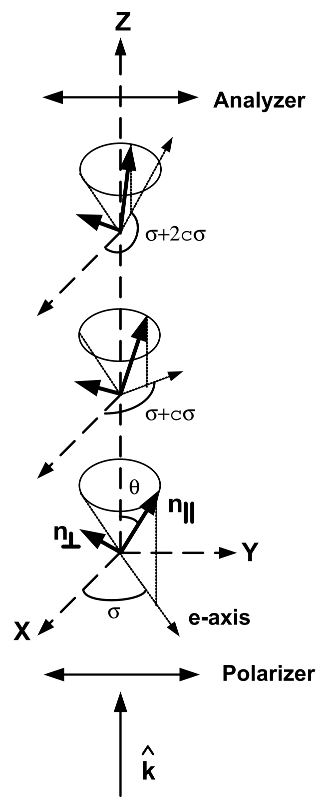

Fig. 1. Schematic of anisotropic plates oriented successively with a twist angle $c\sigma$ between two successive layers. The whole structure is between two parallel polarizers.

the jth layer is then given by $\phi_j = ((j-1)c+1)\sigma$. This form represents a general linearly twisted structure, which can be seen from the fact that if the first layer is oriented at azimuth angle σ and the successive twist between layers is $\delta\phi$, then the azimuth of the jth layer is $\phi_j = (j-1)\delta\phi + \sigma$, which when compared to the above form gives $c = \delta\phi/\sigma$. The special cases of fan Solc structure and spontaneously twisted case are obtained when $c = 2$ and $c = 1$, respectively. The twist angle of the last layer is given by $\phi_N = ((N-1)c+1)\sigma$ with N being the total number of layers. Using the same notation in the well known book of Yariv and Yeh [2], the Jones matrix for the jth plate in the xyz system is given by

$$J_{xyz} = R(-\phi_j)J_0R(\phi_j) \tag{1}$$

where $R(\phi_j)$ is the rotation matrix and J_0 is the Jones matrix in the system of the principal axis of the plate given by

$$J_0 = \exp(-\mathrm{i}\Gamma_{\mathrm{av}})\begin{pmatrix} \exp(-\mathrm{i}\Gamma/2) & 0 \\ 0 & \exp(\mathrm{i}\Gamma/2) \end{pmatrix} \tag{2}$$

The phase retardation is given by $\Gamma = 2\pi d\Delta n/\lambda$ where $\Delta n = n_{\mathrm{s}} - n_{\mathrm{f}}$ is the birefringence and $\Gamma_{\mathrm{av}} = \pi d(n_{\mathrm{s}} + n_{\mathrm{f}})/\lambda$ is the average phase between the fast and slow waves which will be omitted through out the text because it adds only a phase multiplicative factor. The Jones matrix for the whole structure is then given by the ordered product:

$$J = \prod_{j=1}^{N} R(-\phi_{N+1-j})J_0R(\phi_{N+1-j}) \tag{3}$$

Using the fact that $R(\phi_j)R(\phi_{j+1}) = R(\phi_j + \phi_{j+1})$, Eq. (3) can be written as follows:

$$J = R(-\phi_N)[J_0R(c\sigma)]^N R(-(c-1)\sigma) \tag{4}$$

Using Chebyshev's identity for a 2×2 matrix to the power N we get

$$P = [J_0R(c\sigma)]^N = \begin{pmatrix} \frac{\exp(-\mathrm{i}\Gamma/2)\cos(c\sigma)\sin(N\chi) - \sin((N-1)\chi)}{\sin\chi} & -P_{21}^* \\ -\frac{\exp(\mathrm{i}\Gamma/2)\sin(c\sigma)\sin(N\chi)}{\sin\chi} & P_{11}^* \end{pmatrix} \tag{5}$$

where $\cos\chi = \cos(c\sigma)\cos(\Gamma/2)$. The elements of the Jones matrix for the whole structure are given by

$$J_{11} = (P_{11}\cos\phi_N - P_{21}\sin\phi_N)\cos[(c-1)\sigma] - (P_{11}^*\sin\phi_N + P_{21}^*\cos\phi_N)\sin[(c-1)\sigma] \tag{6}$$

$$J_{12} = -(P_{11}\sin\phi_N + P_{21}^*\cos\phi_N)\cos[(c-1)\sigma] - (P_{11}\cos\phi_N - P_{21}\sin\phi_N)\sin[(c-1)\sigma] \tag{7}$$

$J_{21} = -J_{12}^*$ and $J_{22} = J_{11}^*$, where P_{ij} with $i,j = 1,2$ are the matrix elements of the matrix P given in Eq. (5). The transmittance between parallel and crossed polarizers is given by $T_{\mathrm{pp}} = |J_{11}|^2$ and $T_{\mathrm{ps}} = |J_{12}|^2$, respectively. These two equations can even be explicitly written as

$$J_{11} = \frac{\sin(N\chi)[\cos\phi_N\cos\sigma\exp(-\mathrm{i}\Gamma/2) + \sin\phi_N\sin\sigma\exp(\mathrm{i}\Gamma/2)] - \cos(N\sigma)\sin((N-1)\chi)}{\sin\chi} \tag{8}$$

$$J_{12} = \frac{\sin(N\chi)[\cos\phi_N\sin\sigma\exp(-\mathrm{i}\Gamma/2) - \sin\phi_N\cos\sigma\exp(\mathrm{i}\Gamma/2)] + \sin(N\sigma)\sin((N-1)\chi)}{\sin\chi} \tag{9}$$

It should be noted that the choice of c is not completely arbitrary as it is limited by the azimuth of the first layer, by the final azimuth and by the number of layers. It has to satisfy the relation

$$c = (\phi_N - \sigma)/(\sigma(N-1)) \tag{10}$$

Equivalently one can say that the successive twist is determined from the relation $c\sigma = (\phi_N - \sigma)/(N-1)$, which for small σ it is determined mainly by the final twist and the number of layers. However, for small number of layers, the initial twist becomes large affecting the determination of the successive twist as well.