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Linear calibration procedure for the phase-to-height relationship in phase measurement profilometry

Paulo J. Tavares [a,*], Mário A. Vaz [b]

[a] Prudente & Tavares, Technical Consulting, Edifício Península, E# 305, Praça do Bom Sucesso 127, 4150-146 Porto, Portugal
[b] Mechanical Engineering Department, Engineering Faculty, Porto University, Portugal

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Abstract

Calibration of the relationship between height and phase is of uttermost importance to perform accurate 3D measurements in phase measurement profilometry. This work reports a different approach to this problem by first looking at the analytical expression for this relationship and determining the regime spanned by the fringe analysis method. The conclusions thus ascertained, amply justify confronting the analytical expression with a simple normalization procedure of the experimental data, with a remarkable matching between both results. In light of this, a linear calibration procedure with just one plane is proposed and verified experimentally.

Keywords: Three-dimensional measurement; Shape measurement; Optical metrology; Profilometry; Fringe projection; Structured light

1. Introduction

Accurate non-contact three-dimensional measurements have drastically influenced physical sensing applications in many areas of knowledge, from industrial production to health care, artwork inspection, robot vision and many other engineering or science activities.

Phase measurement profilometry (PMP), be it phase shift [1] or Fourier transform [2–5], is certainly amongst the most successful three-dimensional active vision techniques available today.

These procedures calculate phase shift mappings of surface deformations or profile-to-reference differences and rely, at some point, on the translation of the obtained phase map to either a dislocation or a height measurement. The relation between phase and height has thus been the subject of a large number of works over the years, due to its impact on the overall method's accuracy. Both analytical [3,6–9] and empirical [10–14] approaches were reported to present days, although an explicit comparison of both trends has not yet been published, to the authors knowledge.

Analytical methods, simple to apply as they may be, rely on a precise determination of the camera and projector locations and do not explicitly account for the distortions of both projection and imaging optics. The latter can become a major issue, in those cases where the aberrations are either unidentified or known to severely impact the final result. The former argument can also be decisive to final accuracy, when even the smallest change between projector and camera positions can result in erroneous measurements.

On the other hand, most of the phase-to-depth empirical calibration methods proposed to date interpolate the results from a set of reference plane dislocations, for each of which a phase map has been calculated. The accuracy of this procedure depends on the precision of the travelling stage that carries the reference plane and is fairly awkward to manoeuvre outside of laboratory installations.

The work depicted herein takes a different view of this issue by looking at the analytical phase-to-depth expression and determining the region of interest along that curve.

* Corresponding author. Tel.: +351 22 508 2151; fax: +351 22 508 1584.
E-mail addresses: jtavares@fe.up.pt (P.J. Tavares), gmavaz@fe.up.pt (M.A. Vaz).

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The choice of a linear calibration procedure is then strongly supported by the evidence that, for the most part of the plausible working regime, this relation is indeed a linear one. The striking match of the experimental and the analytical results is but another confirmation of these findings, which ultimately led to devising a single image calibration procedure, described and analyzed at the end of the note.

2. Analytical phase-to-depth relation

Fig. 1 depicts a typical optical crossed axis phase measurement profilometry setup. A fringe pattern with a known spatial frequency is projected onto the object under test by projector P and the image is captured at camera C. E_p and E_c refer to the nodal points of the projection and imaging systems, respectively. The projection and imaging systems optical axis cross at point O on a reference plane R_0. L_0 is the distance from the camera – projector plane to the reference plane, d is the distance between centers of projection, θ is the angle between projector and camera and s represents the intersection of E_pB with the object surface.

E_tD represents a telecentric projection through s, adequately described by

$$g_T(x,y) = a(x,y) + b(x,y)\cos[2\pi f_0 x] \tag{1}$$

where $a(x,y)$ and $b(x,y)$ represent non-uniform distributions of reflectivity on the surface of the object and f_0 is the fundamental frequency of the observed grating image.

In the non-telecentric case, the pattern on the reference plane can then be described by

$$g_0(x,y) = a(x,y) + b(x,y)\cos[2\pi f_0 x + \phi_0(x)] \tag{2}$$

with

$$\phi_0(x) = 2\pi f_0 \overline{BD} \tag{3}$$

When the object is in place, the ray from E_p that strikes the reference plane at B is seen from the camera as coming from A and the phase expression for $h(x,y) \neq 0$ is therefore

$$\phi(x,y) = 2\pi f_0 \overline{AD} \tag{4}$$

The fringe pattern on the object surface is expressed by

$$g_s(x,y) = a(x,y) + b(x,y)\cos[2\pi f_0 x + \phi(x,y)] \tag{5}$$

and the phase difference between reference plane and object surface is then

$$\Delta\phi(x,y) = 2\pi f_0(\overline{AD} - \overline{BD}) = 2\pi f_0 \overline{AB} \tag{6}$$

As shown in [3], this phase difference is completely recovered as

$$\Delta\phi(x,y) = \mathrm{Im}\{\log[G_s(u,v)G_0^*(u,v)]\} \tag{7}$$

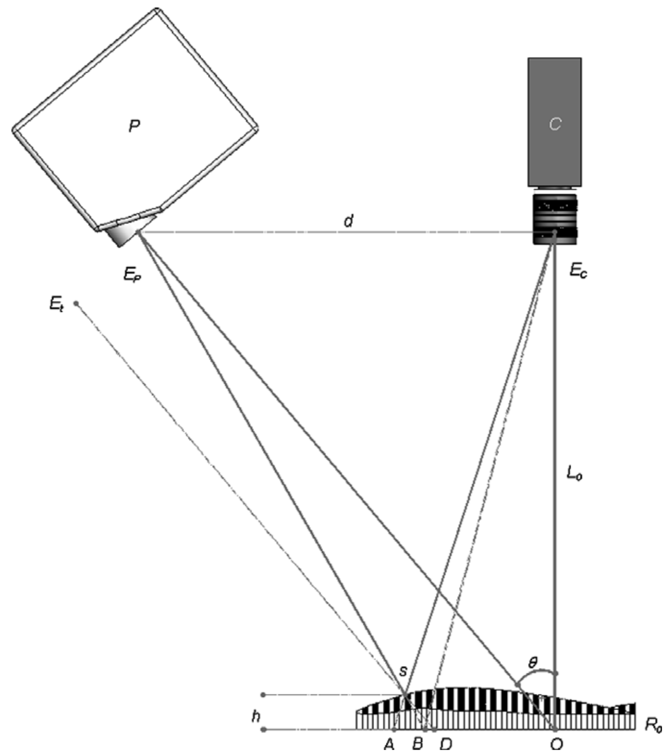

Fig. 1. Fourier transform profilometry crossed axis setup.

where $G(u,v)$ are the Fourier Transforms of the fringe patterns and $*$ denotes complex conjugation.

In order to establish a relation between phase and height one uses the fact triangles E_psE_c and AsB are similar

$$AB = \frac{dh}{L_0 - h} \tag{8}$$

so

$$\Delta\phi(x,y) = 2\pi f_0 AB \Rightarrow h(x,y) = \frac{L_0\Delta\phi(x,y)}{\Delta\phi(x,y) + 2\pi f_0 d} \tag{9}$$

A thorough analysis in [9] indicates $h(x,y)$ is in fact a function of the lateral coordinate x and the angle between projector and camera θ as

$$h(x,y) = \frac{L_0}{\dfrac{2\pi L_0^2 d\cos\theta}{P_0\Delta\phi(x,y)(L_0 + x\cos\theta\sin\theta)^2} - \dfrac{d\cos\theta\sin\theta}{L_0 + x\cos\theta\sin\theta} + 1} \tag{10}$$

with

$$P_0 = \frac{1}{f_0} = \frac{P}{\cos\theta} \tag{11}$$

P being the projected grating period.

Now, an analysis of either expression for $h(x,y)$ will reveal they are not that much different for a plausible application regime. Reversing the last function for $h(x,y)$ above gives

$$\Delta\phi = \frac{2h\pi L_0^2 d\cos\theta}{P_0(h(dL_0\cos\theta\sin\theta + dx\cos^2\theta\sin^2\theta - L_0^2) + L_0^3 + (L_0 - h)x\cos\theta\sin\theta(2L_0 + x\cos\theta\sin\theta))} \tag{12}$$