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Convergence of general beams into Gaussian intensity profiles after propagation in turbulent atmosphere

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Abstract

It is shown that a general shaped laser beam will eventually approach a Gaussian average intensity profile after propagation in turbulent atmosphere. In our formulation, source field at the exit plane of the laser is taken as the product of arbitrary functions of source transverse coordinates with Gaussian exponential modulations. Following the expansion of the arbitrary functions in terms of Hermite polynomials, the average receiver intensity expression is derived using the extended Huygens–Fresnel principle and the conditions for the intensity profile to assume a Gaussian shape are stated. The results are illustrated by simulating various source field distributions.

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1. Introduction

Propagation characteristics of various types of laser beam profiles in free space are being investigated. In this context, the general class of Hermite-sinusoidal-Gaussian laser beams [1,2], and the subclasses such as cosh-Gaussian [3], elegant Hermite-cosh-Gaussian [4], Hermite-cosine-Gaussian [5], and off-axial Hermite-cosh-Gaussian [6] laser beams are studied in detail. Numerous other types like flat-topped multi-Gaussian [7] and Bessel–Gauss and Laguerre–Gauss laser beams [8] are also reported. During our recent works [9–13], in which we examined the propagation characteristics of various special forms of the general Hermite-sinusoidal-Gaussian and higher order annular laser beams in atmospheric turbulence, we constantly observed that after having propagated, these beams have a tendency to approach Gaussian shape, irrespective of the source field excitation. The purpose of this article is to explore whether such an attribution is also shared by a general shaped laser beam, and if so, under which conditions.

Concerning the subject of eventual irradiance pattern in propagation, some studies appeared in the literature, particularly at the beginning of 1970s, with majority of them being based on simple source excitation types. For instance, the formulation of a beam remaining Gaussian during propagation in a turbulent medium, given a Gaussian initial excitation, and the associated conditions, were stated by Prokhorov et al. [14]. On the other hand, Whitman and Beran [15] found that, with certain assumptions, the intensity profile of an arbitrary cylindrically symmetric source beam would be Gaussian in the limit

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of propagation length being extended to infinity. Lutomirski and Yura [16] presented the case of plane wave in a circular aperture (essentially flat topped beam), deriving the profile shaping functions applicable to different propagation ranges.

For our present analysis, the source field expression is taken as the product of two independent arbitrary polynomial functions of source transverse coordinates x and y, accompanied by the two separate Gaussian exponentials to ensure the physical realization of the beam type used in this paper. Consequently, the beam profile is configured with complete $x-y$ asymmetry and general shaping, as a result making our model the most general case of previous studies.

2. Formulation

We stipulate a propagation geometry consisting of source and receiver planes both positioned in a perpendicular manner to the axis of propagation, along which turbulent atmosphere exists. In this setup, a laser emitting a beam of general shape is placed around the origin of the exit, i.e., source plane. For the general beam to convey a finite amount of energy and thus to be realizable, the shaping functions are assumed square integrable and an exponential Gaussian windowing profile is superimposed on this general pattern. Based on the conventions of our previous studies [9–13] and assuming a collimated beam, where the radius of phase front goes to infinity, the field at the exit plane of the laser ($z = 0$) will then be given by

$$u(\mathbf{s}, z=0) = \exp(-0.5 s_x^2/\alpha_{sx}^2) f(a_x s_x) \exp(-0.5 s_y^2/\alpha_{sy}^2) g(a_y s_y) \tag{1}$$

where $\mathbf{s} = (s_x, s_y)$ is the transverse coordinate at the laser exit plane, z is the propagation axis, α_{sx} and a_x, α_{sy}, and a_y are the factors denoting the scaling of the beam related to source dimensions in x, y directions, respectively, and f and g are arbitrary functions such that

$$\int_{-\infty}^{\infty} \exp(-a_x^2 s_x^2) f^2(a_x s_x) \mathrm{d}s_x < \infty$$
$$\int_{-\infty}^{\infty} \exp(-a_y^2 s_y^2) g^2(a_y s_y) \mathrm{d}s_y < \infty \tag{2}$$

Note that the exponentials appearing in Eq. (2) are the weight functions having no association to the exponentials of Eq. (1). Under the conditions given in Eq. (2), f and g can be expanded in terms of Hermite polynomials as

$$f(a_x s_x) = \sum_{n=0}^{\infty} c_n H_n(a_x s_x) \tag{3}$$

$$g(a_y s_y) = \sum_{m=0}^{\infty} d_m H_m(a_y s_y) \tag{4}$$

where

$$c_n = a_x\{1/[2^n(n!)(\pi)^{1/2}]\} \int_{-\infty}^{\infty} \exp(-a_x^2 s_x^2) f(a_x s_x) H_n(a_x s_x) \mathrm{d}s_x \tag{5}$$

$$d_m = a_y\{1/[2^m(m!)(\pi)^{1/2}]\} \int_{-\infty}^{\infty} \exp(-a_y^2 s_y^2) g(a_y s_y) H_m(a_y s_y) \mathrm{d}s_y \tag{6}$$

In Eqs. (5) and (6), ! means the factorial notation. The intensity at the source plane is obtained from

$$I(\mathbf{s}) = u(\mathbf{s}, z=0)[u(\mathbf{s}, z=0)]^* \tag{7}$$

At this stage, for completeness and clarity, we wish to specify the status of Eq. (1) in relation to our previous formulations. In our earlier works [9–13], the shaping functions, f and g used to be single Hermite polynomials, whereas in this present study, they are arbitrary functions that may be chosen at liberty bearing in mind the condition governed by Eq. (2). Quite a broad range of functions are able to satisfy Eq. (2) and there exist orthogonal polynomials other than Hermite polynomials for expansions of f and g functions. [17]. The current study employs Hermite polynomials for merely mathematical convenience.

Applying the extended Huygens–Fresnel principle for turbulent atmosphere, the average intensity at a propagation distance $z = L$ is formulated as [9–13]

$$\begin{aligned}\langle I(p, L)\rangle = k^2/(2\pi L)^2 &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{d}^2\mathbf{s}_1\,\mathrm{d}^2\mathbf{s}_2 u(\mathbf{s}_1) u^*(\mathbf{s}_2) \\ &\times \exp\{\mathrm{i}k[(\mathbf{p}-\mathbf{s}_1)^2 - (\mathbf{p}-\mathbf{s}_2)^2]/(2L)\} \\ &\times \langle \exp[\psi(\mathbf{s}_1, \mathbf{p}) + \psi^*(\mathbf{s}_2, \mathbf{p})]\rangle \end{aligned} \tag{8}$$

Via the integral in Eq. (8), Huygens–Fresnel principle serves to find the intensity on the receiver plane, by applying the response of the propagation environment on the field of excitation. In this sense, the exponential appearing in the second line of Eq. (8) expresses the diffraction that the propagating beam is subjected to, which is also common to free space propagation [18]. The third line of Eq. (8), on the other hand, describes the effects of turbulence in propagation. Turning to the definitions of particular terms in Eq. (8); $\mathbf{p} = (p_x, p_y)$ denotes the transverse coordinate at the receiver plane ($z = L$), $\mathrm{i} = \sqrt{-1}$, k is the wave number, $\psi(\mathbf{s}, \mathbf{p})$ is the solution to Rytov method representing the random part of the complex phase of a spherical wave propagating from the source point ($\mathbf{s}, z = 0$) to the receiver point ($\mathbf{p}, z = L$), and $\langle\ \rangle$ indicates the ensemble average over the turbulent medium statistics, and for the last line of Eq. (8) is given by

$$\begin{aligned}\langle \exp[\psi(\mathbf{s}_1, \mathbf{p}) + \psi^*(\mathbf{s}_2, \mathbf{p})]\rangle &= \exp[-0.5 D_\psi(\mathbf{s}_1 - \mathbf{s}_2)] \\ &\cong \exp[-\rho_0^{-2}(\mathbf{s}_1 - \mathbf{s}_2)^2]\end{aligned} \tag{9}$$

where $D_\psi(\mathbf{s}_1 - \mathbf{s}_2)$ refers to the wave structure function, and $\rho_0 = (0.545 C_n^2 k^2 L)^{-3/5}$ is the coherence length of a spherical wave propagating in the turbulent medium, C_n^2 is the structure constant. The setting in Eq. (9) and the subsequent definition of ρ_0 imply that we have approximated the actual five thirds power of $(\mathbf{s}_1 - \mathbf{s}_2)$ to quadratic, additionally Kolmogorov spectrum has been employed [19]. Substituting Eqs. (1), (3), (4), and (9) into Eq. (8) and using Eq.