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Vectorial structures of non-paraxial linearly polarized Gaussian beam and their beam propagation factors

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Abstract

Based on the vectorial structure of non-paraxial electromagnetic beam and non-paraxial vectorial moment theory, the relationship of the beam waists, the divergence angles and the beam propagation factors among non-paraxial linearly polarized Gaussian beam, its TE and TM terms have been presented, respectively. The analytical beam propagation factors are given and further discussed at the highly non-paraxial case. The maximum divergence angles in the x-direction of non-paraxial linearly polarized Gaussian beam, its TE and TM terms are all 54.7°, and those in the y-direction are limited to be 63.4°, 67.7° and 39.2°, respectively. As TE and TM terms are orthogonal and can be detached at the far field, the potential applications of the isolated TE and TM terms are deserved further investigation.

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1. Introduction

With the advent of many new mini-dimensional optical structures and laser light sources, the description of optical propagation in the non-paraxial region receives considerable interest [1–10]. As the transverse spot size of non-paraxial laser beam is smaller than or of the order of the light wavelength, the paraxial approximation fails [11]. Due to the electromagnetic vector effect, moreover, scalar description is no longer applicable to non-paraxial laser beam. Therefore, non-paraxial laser beam must be accurately described by the solution of the Maxwell equations. The full vector angular spectrum method is a useful tool to resolve the Maxwell equations [12]. A representation of general electric solution of the Maxwell equations can be essentially expressed as a sum of two terms. One is TE term with the electric field transverse to the propagating axis, and the other is TM term with the associated magnetic filed transverse to the propagating axis [13]. According to this principle, non-paraxial linearly polarized Gaussian beam, the rigorous solution of Maxwell's equations for a confocal resonator [2,14], can also be divided into TE and TM terms. As TE and TM terms are orthogonal to each other at the far field, they can be detached at the far field. Moreover, most practical applications of laser beams are involved in the far field. Therefore, the respective propagation characteristics of the TE and TM terms are still worthy of investigation, though the propagation of non-paraxial linearly polarized Gaussian beam has been studied as an integrated beam [10]. Moreover, the relationship of the beam propagation factors among the whole beam, its TE and TM terms is investigated in the rest of this paper.

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The paper is arranged as follows. In the next section, the analytical formulae of the beam waists, the far field divergence angles and the beam propagation factors are presented on the basis of vectorial structure of non-paraxial electromagnetic beam and non-paraxial vectorial moment theory [15]. The analytical formulae are further discussed at the highly non-paraxial case in Section 3. Finally, the main conclusions are outlined in Section 4.

2. Theoretical analysis

In Cartesian coordinate system, the z-coordinate is taken to be the propagating axis. The half space $z \geqslant 0$ is filled with a homogeneous, isotropic, non-conducting and transparent medium with electric permittivity ε and magnetic permeability μ. A non-paraxial linearly polarized Gaussian beam, the rigorous solution of Maxwell's equations for a confocal resonator, results from the Gaussian boundary condition of the Hertz vector [2,14]. According to the vectorial structure of non-paraxial electromagnetic beam [13], the electric field of non-paraxial linearly polarized Gaussian beam propagating toward half space $z \geqslant 0$ can be divided into two terms

$$\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}_{\mathrm{TE}}(\boldsymbol{r}) + \boldsymbol{E}_{\mathrm{TM}}(\boldsymbol{r}), \tag{1}$$

where $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ is the displacement vector. Here, TE and TM denote that the z component of electric and magnetic fields is zero, respectively. While, TE and TM elsewhere generally mean that the x or y component of electric and magnetic fields is zero, respectively. Taken the evanescent waves into consideration, $\boldsymbol{E}_{\mathrm{TE}}$ and $\boldsymbol{E}_{\mathrm{TM}}$ can be expressed in the plane wave angular spectrum representation as

$$\begin{pmatrix} \boldsymbol{E}_{\mathrm{TE}}(\boldsymbol{r}) \\ \boldsymbol{E}_{\mathrm{TM}}(\boldsymbol{r}) \end{pmatrix} = \int_0^{\infty} \int_0^{2\pi} \begin{pmatrix} \boldsymbol{A}_{\mathrm{TE}}^{\mathrm{E}} \\ \boldsymbol{A}_{\mathrm{TM}}^{\mathrm{E}} \end{pmatrix} \exp(ik\boldsymbol{r}\cdot\boldsymbol{s})\rho\,\mathrm{d}\rho\,\mathrm{d}\varphi, \tag{2}$$

$$\text{with} \quad \begin{pmatrix} \boldsymbol{A}_{\mathrm{TE}}^{\mathrm{E}} \\ \boldsymbol{A}_{\mathrm{TM}}^{\mathrm{E}} \end{pmatrix} = \frac{c}{2\pi} \exp\left(-\frac{c\rho^2}{2}\right) \begin{pmatrix} \sin\varphi \boldsymbol{e}_1 \\ \gamma\cos\varphi \boldsymbol{e}_2 \end{pmatrix}, \tag{3}$$

where $\gamma = \sqrt{1-\rho^2}$, and $c = \frac{1}{2}k^2w_0^2$ with w_0 the initial Gaussian half width. $k = \frac{2\pi}{\lambda}$ is the wave number. $\lambda = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}}$ is the wavelength in the medium. ω is the circular frequency. The dot denotes scalar product. The three unit vectors are defined as follows:

$$\boldsymbol{s} = \rho\cos\varphi\boldsymbol{i} + \rho\sin\varphi\boldsymbol{j} + \gamma\boldsymbol{k}, \qquad \boldsymbol{e}_1 = \sin\varphi\boldsymbol{i} - \cos\varphi\boldsymbol{j}, \quad \boldsymbol{e}_2 = \gamma\cos\varphi\boldsymbol{i} + \gamma\sin\varphi\boldsymbol{j} - \rho\boldsymbol{k}. \tag{4}$$

They form a mutually perpendicular right-handed system

$$\boldsymbol{s}\times\boldsymbol{e}_1 = \boldsymbol{e}_2, \quad \boldsymbol{e}_1\times\boldsymbol{e}_2 = \boldsymbol{s}, \quad \boldsymbol{e}_2\times\boldsymbol{s} = \boldsymbol{e}_1. \tag{5}$$

where $\rho\cos\varphi/\lambda$ and $\rho\sin\varphi/\lambda$ are the transversal spatial frequencies. γ/λ is the longitudinal spatial frequency. The physics meaning of ρ is that values of $\rho < 1$ correspond to homogeneous plane waves propagating at angles $\sin^{-1}\rho$ with respect to the z-axis, whereas values of $\rho > 1$ evanescent waves.

Similarly, the corresponding magnetic field of non-paraxial linearly polarized Gaussian beam can also be expressed as a sum of two terms

$$\boldsymbol{H}(\boldsymbol{r}) = \boldsymbol{H}_{\mathrm{TE}}(\boldsymbol{r}) + \boldsymbol{H}_{\mathrm{TM}}(\boldsymbol{r}). \tag{6}$$

$\boldsymbol{H}_{\mathrm{TE}}$ and $\boldsymbol{H}_{\mathrm{TM}}$ can be written in the plane wave angular spectrum form

$$\begin{pmatrix} \boldsymbol{H}_{\mathrm{TE}}(\boldsymbol{r}) \\ \boldsymbol{H}_{\mathrm{TM}}(\boldsymbol{r}) \end{pmatrix} = \int_0^{\infty} \int_0^{2\pi} \begin{pmatrix} \boldsymbol{A}_{\mathrm{TE}}^{\mathrm{H}} \\ \boldsymbol{A}_{\mathrm{TM}}^{\mathrm{H}} \end{pmatrix} \exp(\mathrm{i}k\boldsymbol{r}\cdot\boldsymbol{s})\rho\,\mathrm{d}\rho\,\mathrm{d}\varphi, \tag{7}$$

$$\text{with} \quad \begin{pmatrix} \boldsymbol{A}_{\mathrm{TE}}^{\mathrm{H}} \\ \boldsymbol{A}_{\mathrm{TM}}^{\mathrm{H}} \end{pmatrix} = \sqrt{\frac{\varepsilon}{\mu}}\frac{c}{2\pi} \exp\left(-\frac{c\rho^2}{2}\right) \begin{pmatrix} \sin\varphi \boldsymbol{e}_2 \\ -\gamma\cos\varphi \boldsymbol{e}_1 \end{pmatrix}. \tag{8}$$

The time dependent factor $\exp(-\mathrm{i}\omega t)$ is omitted in Eqs. (2) and (7). The magnetic field of non-paraxial linearly polarized Gaussian beam turns out to be

$$\boldsymbol{H}(\boldsymbol{r}) = \int_0^{\infty} \int_0^{2\pi} \sqrt{\frac{\varepsilon}{\mu}}\frac{c}{2\pi} \exp\left(-\frac{c\rho^2}{2}\right) (\gamma\boldsymbol{j} - \rho\sin\varphi\boldsymbol{k}) \exp(\mathrm{i}k\boldsymbol{r}\cdot\boldsymbol{s})\rho\,\mathrm{d}\rho\,\mathrm{d}\varphi. \tag{9}$$

As the x component of magnetic field is equal to zero, non-paraxial linearly polarized Gaussian beam in this paper is also called as TM Gaussian beam [10]. To avoid confusion, non-paraxial linearly polarized Gaussian beam is adopted here just as Ref. [13]