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A degenerate three-level laser with a parametric amplifier

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Abstract

The aim of this paper is to study the squeezing and statistical properties of the light produced by a degenerate three-level laser whose cavity contains a degenerate parametric amplifier. In this quantum optical system the top and bottom levels of the three-level atoms injected into the laser cavity are coupled by the pump mode emerging from the parametric amplifier. For a linear gain coefficient of 100 and for a cavity damping constant of 0.8, the maximum intracavity squeezing is found at steady state and at threshold to be 93%.

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Keywords: Quadrature variance; Squeezing spectrum; Photon statistics

1. Introduction

There has been a considerable interest in the analysis of the quantum properties of the squeezed light generated by various quantum optical systems [1–9]. In squeezed light the fluctuations in one quadrature is below the vacuum level at the expense of enhanced fluctuations in the other quadrature, with the product of the uncertainties in the two quadratures satisfying the uncertainty relation. In addition to exhibiting a nonclassical feature, squeezed light has potential applications in precision measurements and noiseless communications [10,11].

Some authors have studied the squeezing and statistical properties of the light produced by three-level lasers when either the atoms are initially prepared in a coherent superposition of the top and bottom levels [12–14] or when these levels are coupled by a strong coherent light [13]. These studies show that a three-level laser can under certain conditions generate squeezed light. In such a laser, three-level atoms in a cascade configuration are injected at a constant rate into the cavity coupled to a vacuum reservoir via a single-port mirror. When a three-level atom makes a transition from the top to bottom level via the intermediate level, two photons are generated. The two photons are highly correlated and this correlation is responsible for the squeezing of the light produced by a three-level laser. On the other hand, it is well known that a parametric oscillator is a typical source of squeezed light [2–6], with a maximum intracavity squeezing of 50%. Recently Fesseha [12] has studied a three-level laser with a parametric amplifier in which three-level atoms, initially prepared in a coherent superposition of the top and bottom levels, are injected into the cavity. He has found that the effect of the parametric amplifier is to increase the intracavity squeezing by a maximum of 50%.

In this paper we consider a degenerate three-level laser whose cavity contains a degenerate parametric amplifier (DPA) and coupled to a vacuum reservoir. The top and bottom levels of the three-level atoms injected into the cavity are coupled by the pump mode emerging from the parametric amplifier. And the three-level atoms are initially prepared in such a way that the probabilities of finding the atoms at the top and bottom levels are equal. We expect that a highly squeezed light can be generated by

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the quantum optical system under consideration. Thus our interest is to analyze the squeezing and statistical properties of the light generated by this system.

We obtain, applying the master equation, stochastic differential equations for the cavity mode variables associated with the normal ordering. The solutions of the resulting equations are used to determine the quadrature variance, the squeezing spectrum, and the mean photon number. Moreover, applying the same solutions, we determine the antinormally ordered characteristic function with the aid of which the Q function is obtained. Then the Q function is used to calculate the photon number distribution.

2. Stochastic differential equations

Three-level atoms in a cascade configuration are injected into the laser cavity at a constant rate r_a and removed from the cavity after a certain time τ. We represent the top, middle, and bottom levels of a three-level atom by $|a\rangle$, $|b\rangle$ and $|c\rangle$, respectively. We assume the transitions between levels $|a\rangle$ and $|b\rangle$ and between levels $|b\rangle$ and $|c\rangle$ to be dipole allowed, with direct transitions between levels $|a\rangle$ and $|c\rangle$ to be dipole forbidden. We consider the case for which the cavity mode is at resonance with the two transitions $|a\rangle \rightarrow |b\rangle$ and $|b\rangle \rightarrow |c\rangle$ (see Fig. 1).

The Hamiltonian describing the coupling of levels $|a\rangle$ and $|c\rangle$ by the pump mode emerging from the parametric amplifier can be expressed as

$$\hat{H}' = \mathrm{i}\frac{\Omega}{2}(|c\rangle\langle a| - |a\rangle\langle c|), \tag{1}$$

in which $\Omega = 2g'\mu$ with g' and μ being respectively the coupling constant and the amplitude of the pump mode. In addition, the interaction of a three-level atom with the cavity mode can be described by the Hamiltonian

$$\hat{H}'' = \mathrm{i}g[\hat{a}^\dagger(|b\rangle\langle a| + |c\rangle\langle b|) - \hat{a}(|a\rangle\langle b| + |b\rangle\langle c|)], \tag{2}$$

where g is the coupling constant and $\hat{a}$ is the annihilation operator for the cavity mode. Thus the Hamiltonian describing the interaction of a three-level atom with the cavity mode and with the pump mode emerging from the parametric amplifier has the form

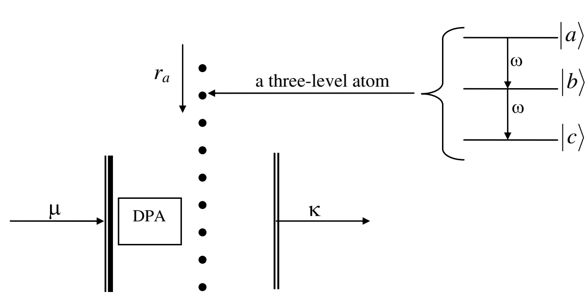

Fig. 1. A degenerate three-level laser with a degenerate parametric amplifier.

$$\hat{H} = \mathrm{i}g[\hat{a}^\dagger(|b\rangle\langle a| + |c\rangle\langle b|) - \hat{a}(|a\rangle\langle b| + |b\rangle\langle c|)] + \mathrm{i}\frac{\Omega}{2}(|c\rangle\langle a| - |a\rangle\langle c|). \tag{3}$$

We take the initial state of a single three-level atom to be

$$|\psi_A(0)\rangle = \frac{1}{\sqrt{2}}|a\rangle + \frac{1}{\sqrt{2}}|c\rangle \tag{4}$$

and hence the density operator for a single atom is

$$\hat{\rho}_A(0) = \frac{1}{2}|a\rangle\langle a| + \frac{1}{2}|a\rangle\langle c| + \frac{1}{2}|c\rangle\langle a| + \frac{1}{2}|c\rangle\langle c|. \tag{5}$$

It can be readily established that the equation of evolution of the density operator for the laser cavity mode, coupled to a vacuum reservoir, has in the linear and adiabatic approximation the form [15]

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho} = R(2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}\hat{a}^\dagger\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^\dagger) + S(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}) + U(\hat{a}^\dagger\hat{\rho}\hat{a}^\dagger + \hat{a}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}^{\dagger 2} - \hat{a}^2\hat{\rho}) + V(\hat{a}^\dagger\hat{\rho}\hat{a}^\dagger + \hat{a}\hat{\rho}\hat{a} - \rho\hat{a}^2 - \hat{a}^{\dagger 2}\hat{\rho}), \tag{6}$$

where

$$R = \frac{A}{4B}\left[1 - \frac{3\beta}{2} + \beta^2\right], \tag{7a}$$

$$S = \frac{A}{4B}\left[\frac{2\kappa B}{A} + 1 + \frac{3\beta}{2} + \beta^2\right], \tag{7b}$$

$$U = \frac{A}{4B}\left[-1 + \frac{\beta}{2} + \frac{\beta^2}{2} + \frac{\beta^3}{2}\right], \tag{7c}$$

$$V = \frac{A}{4B}\left[-1 - \frac{\beta}{2} + \frac{\beta^2}{2} - \frac{\beta^3}{2}\right], \tag{7d}$$

$$B = (1 + \beta^2)(1 + \frac{\beta^2}{4}), \tag{7e}$$

$$\beta = \Omega/\gamma, \tag{7f}$$

$$A = \frac{2g^2 r_a}{\gamma^2} \tag{8}$$

is the linear gain coefficient, κ is the cavity damping constant, and γ is the atomic decay rate assumed to be the same for all the three levels.

Moreover, a degenerate parametric amplifier with the pump mode treated classically is describable in the interaction picture by the Hamiltonian

$$\hat{H} = \frac{\mathrm{i}\varepsilon}{2}(\hat{a}^{\dagger 2} - \hat{a}^2), \tag{9}$$

in which $\varepsilon = \lambda\mu$ with λ being the coupling constant. The master equation associated with this Hamiltonian has the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho} = \frac{\varepsilon}{2}(\hat{\rho}\hat{a}^2 - \hat{a}^2\hat{\rho} + \hat{a}^{\dagger 2}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger 2}). \tag{10}$$

Now on account of Eqs. (6) and (10), the master equation for the cavity mode of the quantum optical system under consideration can be written as