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Optics Communications 263 (2006) 1–5
OPTICS COMMUNICATIONS
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Fractal axicons

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Received 20 September 2005; received in revised form 3 January 2006; accepted 12 January 2006

Abstract

Cantor rings are rotational symmetric pupils that are generated from a Cantor set of a given level of growth. These pupils have certain fractal properties. For example, it is known that when illuminated by a general spherical wavefront they provide self-similar patterns at transverse planes in the Fraunhofer region. In this paper, we study the response of Cantor rings when illuminated by a Bessel light beam conforming what we call fractal axicons. It is shown that, with this kind of illumination a close replica of the radial profile of the pupil is obtained along the optical axis, i.e., we show that the axial behaviour of these pupils has self-similarity properties that can be correlated to those of the diffracting aperture. The influence of several construction parameters is numerically investigated.

Keywords: Fractals; Cantor set; Bessel beams; Axial irradiance; Axicons

1. Introduction

Certain natural phenomena exhibit distinctive features that can be associated with the concept of fractal. Fractals are self-similar structures that are invariant under a change of scale which, in addition, have a fractional dimension. Fractals have been known over the past centuries, but certainly it was not until the development of computer science that they became a matter of great interest for scientists in many fields [1].

In optics, the diffraction properties of fractal objects have been studied extensively ranging from simple one-dimensional objects to complex 2D systems (See for example Refs. [2,3] and the references therein), including fractal zone plates [4,5], which allow the fractal focusing of light among other properties. A special case of 2D rotationally symmetric self-similar objects are the Cantor rings (CRs), or Cantor ring diffractals as coined by Jaggard and Jaggard [6]. In their original paper these authors found that the transverse diffraction patterns produced in the far field by CRs exhibit scaling features that are typical of regular fractal structures. Lately the axial behaviour of CRs has also been investigated, but in this case, it has been shown that the irradiance produced by a CR does not present any fractal structure of the aperture [7].

There are still many other interesting features of the CRs that have not been developed but could be significant for future applications. Of particular interest are the diffraction properties given by these fractal structures when illuminated by Bessel light beams. These beams produce a uniform axial distribution with an extremely narrow central peak in the transverse direction [8]. This feature offers, for example, enhanced optical guiding possibilities, which have important restrictions for other types of illumination (in particular, for a Gaussian beam the physical limit is the Rayleigh range). The use of Bessel beams is then optimal for optical trapping and manipulation of microscopic particles and biological cells [9], for optical coherence tomography [10] and more generally for experiments in nonlinear optics [11]. Experimental proposals to obtain nondiffracting Bessel beams are numerous in the literature [12,13], among which we find very reliable the holographic elements [14]

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0030-4018/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.optcom.2006.01.020

and spatial light modulators [15]. The proper combination of these elements with a Cantor rings spatial filter constitutes what we call a fractal axicon, that is, an optical arrangement that focuses light onto the axis with fractal properties.

In this work the axial behaviour of a Bessel beam impinging on a CR is studied. In Section 2, we use the Fresnel diffraction integral to evaluate the axial intensity distribution of an apertured azimuthally-symmetric Bessel beam. We demonstrate that, from the irradiance distribution of the pupil aperture, a nonexact but close replica of it is obtained along the optical axis. Also we give the practical limitations to observe this exclusive behaviour. In Section 3, we present the procedure we followed for the synthesis of the CRs. In Section 4, we perform the numerical analysis of CRs illuminated by a Bessel beam. The influence of the effective Fresnel number [16] associated to a CR on the axial irradiance is investigated. Additionally, the self-similarity of the axial irradiance is compared with the self-similarity of CR itself. Finally, in Section 5 the main results of this paper are outlined and some applications are proposed.

2. Axial intensity distribution of apertured Bessel beams

Bessel beams are solutions of the scalar Helmholtz equation expressed in cylindrical coordinates [7]. They are interpreted as a linear combination of propagating plane waves whose wave vectors $\boldsymbol{k}$ have the same projection $\beta = k\cos\theta$ onto the axis of propagation. In particular for a wave function with azimuthal symmetry, the solely nonsingular solution of the wave equation is given by

$$U_0(r,z) = \exp(jkz\cos\theta)J_0(kr\sin\theta), \tag{1}$$

where J_0 is the Bessel function of first kind and zero-order, and the amplitude on the axis is assumed to be the unity. From Eq. (1), we observe that the wavefield presents an invariant transverse pattern (except for a constant phase) despite of propagation. The beam amplitude at the optical axis is also invariant.

When limited by an aperture, zero-order Bessel beams are no longer free-space diffraction modes and consequently the common diffraction spreading is observed beyond a certain distance. However, in the near field, the transverse amplitude distribution closely resembles a zero-order Bessel function. Additionally, the irradiance along the optical axis is almost unaltered along a certain distance, producing a focal line [17]. From the experimental point of view, the family of optical arrangements which perform an on-axis linear focus are known as axicons [18,19]. Interestingly, it has been stated that the diffraction characteristics of the focal region in refractive axicons are determined by the so-called Fresnel number such as, for example, the ability of generating a nondiffracting Bessel beam and the focal shift of the irradiance maximum, with the consequent reduction of the focal length named focal squeeze [16].

According to the Fresnel–Kirchhoff diffraction integral, the on-axis irradiance distribution of a rotationally symmetric Bessel beam limited by a circular aperture of radius a is given by

$$I(z) = \left(\frac{k}{z}\right)^2 \left| \int_0^a J_0(kr_0\sin\theta)\exp\left(\mathrm{j}\frac{k}{2z}r_0^2\right) r_0\,\mathrm{d}r_0 \right|^2, \tag{2}$$

where z is the axial distance from the aperture to the observation point. Note that under the substitution $\sin\theta = r/z$, the integral in Eq. (2) is analogous to the Fresnel diffraction pattern of a circular aperture illuminated with a uniform plane wave, for which the geometrical approximation in the near field predicts a uniform irradiance distribution for radial values $r \leqslant a$ and zero for $r > a$ [20]. On the analogy to this situation, we can state that the axial irradiance generated by a Bessel beam limited by a circular aperture is constant within the interval

$$z \leqslant z_{\max} = \frac{a}{\sin\theta}, \tag{3}$$

and is zero for values beyond $z_{\max}$. Obviously, this prediction holds when $ka\sin\theta$ tends to infinity, but it is still accurate if $ka\sin\theta$ is several orders of magnitude higher than unity. Following a previous work [16], we may define the Fresnel number for apertured Bessel beams as

$$N = a\sin\theta/\lambda. \tag{4}$$

Accordingly we should impose that N is much higher than the unity.

Let us now consider the irradiance at a given point on the optical axis provided by a rotationally-invariant purely-absorbing pupil with an amplitude transmittance $p(r)$, which is illuminated by a monochromatic nonuniform beam of amplitude $U_0(r,0)$ (see Fig. 1). Within the Fresnel approximation we find that

$$I(z) = \left(\frac{k}{z}\right)^2 \left| \int_0^\infty U_0(r_0,0)p(r_0)\exp\left(\mathrm{j}\frac{k}{2z}r_0^2\right) r_0\,\mathrm{d}r_0 \right|^2. \tag{5}$$

Again, we may use the geometrical approximation under the assumption of very high Fresnel numbers, and we

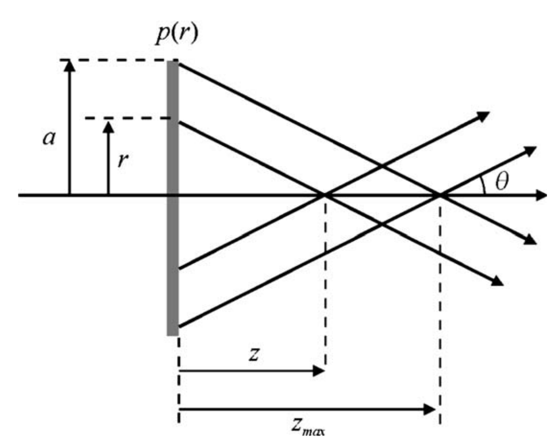

Fig. 1. On-axis irradiance produced by a Bessel beam, of propagating angle θ, impinging on a pupil aperture of transmittance $p(r)$.