

Spatial–angular Mueller matrices

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Abstract

Arbitrary polarization changing devices are described within paraxial approximation by their action on the spatial angular Stokes parameters associated with the generalized rays of a Wigner formulation of wave optics. This approach leads to the introduction of generalized Mueller matrices being functions of the spatial and angular variables parameterizing generalized rays. We apply this approach to a polarization grating.

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1. Introduction

One of the attractive features of Wigner functions is that they allow us to describe rigorously wave phenomena in optics and mechanics by using simple trajectory concepts for light and particle respectively [1–5]. In the optical case to be considered in this work we refer to such effective trajectories as generalized rays.

Concerning polarization, it has been shown that the Wigner formalism allows to introduce Stokes parameters describing the polarization state of generalized light rays [6–9]. Since each ray is represented by an spatial point and a propagation direction, these Stokes parameters depend on spatial and angular variables so we will refer to them as spatial–angular Stokes parameters. Thanks to the angular dependence these parameters provide a complete picture of the polarization of light fields, including its evolution during propagation.

In this work, we study the action of arbitrary polarization changing devices within this formalism for paraxial fields. We find that these actions can be always described by suitably generalized Mueller matrices depending on spa-

tial and angular variables. We shall refer to them as spatial–angular Mueller matrices or Wigner–Mueller matrices. For the sake of illustration, we will particularize this approach to the case of spatially inhomogeneous devices such as a polarization grating [10–12]. In Section 2, we provide the main formulas and concepts required to accomplish these objectives. In Section 3, we derive the main relations for the polarization transformations of generalized rays. In Section 4, we illustrate this approach by applying it to a polarization grating.

2. Spatial–angular Stokes parameters

One of the virtues of Wigner functions is that they accommodate very different situations under a common language. This favors a fruitful exchange of concepts and results between different areas of physics, in particular between classical and quantum optics [2,3,5].

Recently, we have introduced a real, scalar Wigner function $W(\mathbf{r}, \mathbf{p}, \Omega)$ for classical vectorial light fields inspired by parallel results in quantum physics [8]. The polarization state is conveyed by the dependence on $\Omega = (\theta, \phi)$, which is the usual description of the Poincaré sphere in terms of the polar angle θ and the azimuthal angle ϕ . The two-dimensional vector $\mathbf{r}$ represents spatial position in a plane

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perpendicular to the axis z , while the two-dimensional vector $\mathbf{p}$ represents the usual angular representation of the direction of propagation, being the optical counterpart of the quantum momentum operator.

Although the Wigner formalism embodies ray concepts through the joint dependence on $\mathbf{r}$ and $\mathbf{p}$, its validity is not restricted to geometrical optics. On the contrary, it provides an exact and complete representation of wave optics within the limits of applicability of the formalism [6,7,13,14]. The price to be paid is that Wigner functions can take negative values. Because of this, the rays $\mathbf{r}, \mathbf{p}$ are referred to as generalized rays, while those with a negative value of the Wigner function are referred to as dark rays [6,7,13].

The existence of dark rays is required in order to embrace typical wave coherence phenomena, such as interference, within a ray picture. Negative values of the Wigner radiance arise because the simple addition of rays with positive weights (i.e., positive intensity) corresponds exclusively to incoherent beam mixing. However, the theory must include also the coherent superposition of waves, and this forces the appearance of rays with negative weights [6,7,13]. This is closely related with the difficulties encountered by the translation to the wave domain of the concept of radiance [15]. Parallels can be drawn with the Wigner function in quantum mechanics, where the existence of negative values of the Wigner function is an evidence supporting that the quantum theory goes beyond classical physics.

For simplicity, we assume paraxial approximation so that the component of the electric field along axis z is neglected and only the two transversal field components $E_{x,y}(\mathbf{r})$ are necessary. For definiteness, we focus on a single frequency component of a stationary field, so that angle brackets will denote ensemble average. The approach to be developed will account for polarization properties expressible in terms of the cross-spectral density tensor

$$\Gamma_{m,m'}(\mathbf{r}_1, \mathbf{r}_2) = \langle E_m(\mathbf{r}_1) E_{m'}^*(\mathbf{r}_2) \rangle, \quad (1)$$

for $m, m' = x, y$. This correlation tensor in space-frequency representation is usually denoted by $W(\mathbf{r}_1, \mathbf{r}_2, \nu)$, but throughout this work the symbol W will refer to the Wigner function.

The above correlation functions define the spatial Stokes parameters $s_0(\mathbf{r})$, $\vec{s}(\mathbf{r})$ (throughout the upper arrow will refer to three-dimensional vectors)

$$\begin{aligned} s_0(\mathbf{r}) &= \Gamma_{xx}(\mathbf{r}, \mathbf{r}) + \Gamma_{yy}(\mathbf{r}, \mathbf{r}), \\ s_1(\mathbf{r}) &= \Gamma_{xx}(\mathbf{r}, \mathbf{r}) - \Gamma_{yy}(\mathbf{r}, \mathbf{r}), \\ s_2(\mathbf{r}) &= \Gamma_{xy}(\mathbf{r}, \mathbf{r}) + \Gamma_{yx}(\mathbf{r}, \mathbf{r}), \\ s_3(\mathbf{r}) &= i[\Gamma_{xy}(\mathbf{r}, \mathbf{r}) - \Gamma_{yx}(\mathbf{r}, \mathbf{r})], \end{aligned} \quad (2)$$

for which it holds always that

$$\vec{s}^2(\mathbf{r}) \leq s_0^2(\mathbf{r}), \quad s_0(\mathbf{r}) \geq 0. \quad (3)$$

On the other hand, the spatial–angular Stokes parameters for generalized rays are defined as [6–9]

$$\begin{aligned} S_0(\mathbf{r}, \mathbf{p}) &= W_{xx}(\mathbf{r}, \mathbf{p}) + W_{yy}(\mathbf{r}, \mathbf{p}), \\ S_1(\mathbf{r}, \mathbf{p}) &= W_{xx}(\mathbf{r}, \mathbf{p}) - W_{yy}(\mathbf{r}, \mathbf{p}), \\ S_2(\mathbf{r}, \mathbf{p}) &= W_{xy}(\mathbf{r}, \mathbf{p}) + W_{yx}(\mathbf{r}, \mathbf{p}), \\ S_3(\mathbf{r}, \mathbf{p}) &= i[W_{xy}(\mathbf{r}, \mathbf{p}) - W_{yx}(\mathbf{r}, \mathbf{p})], \end{aligned} \quad (4)$$

being $W_{m,m'}(\mathbf{r}, \mathbf{p})$ the elements of the Wigner matrix $W(\mathbf{r}, \mathbf{p})$

$$W_{m,m'}(\mathbf{r}, \mathbf{p}) = \left(\frac{k}{2\pi}\right)^2 \int d^2\mathbf{r}' \Gamma_{m,m'}(\mathbf{r} - \mathbf{r}'/2, \mathbf{r} + \mathbf{r}'/2) e^{ik'\mathbf{p}}, \quad (5)$$

where k is the wave number.

Let us note that in contrast to (3) the potential negativity of the Wigner matrix elements may imply that [9]

$$\vec{S}^2(\mathbf{r}, \mathbf{p}) > S_0^2(\mathbf{r}, \mathbf{p}), \quad S_0(\mathbf{r}, \mathbf{p}) < 0. \quad (6)$$

As discussed above, we will refer to these situations as dark rays.

In order to simplify equations throughout Stokes parameters in boldface characters, such as $\mathbf{s}$, $\mathbf{S}$, and $\tilde{\mathbf{S}}$ (to be defined below), will refer to the four-dimensional vectors

$$\mathbf{s} = \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix}, \quad \tilde{\mathbf{S}} = \begin{pmatrix} \tilde{S}_0 \\ \tilde{S}_1 \\ \tilde{S}_2 \\ \tilde{S}_3 \end{pmatrix}. \quad (7)$$

An expression equivalent to (4) is

$$\mathbf{S}(\mathbf{r}, \mathbf{p}) = \text{tr}[\boldsymbol{\sigma} W(\mathbf{r}, \mathbf{p})], \quad (8)$$

and also

$$W(\mathbf{r}, \mathbf{p}) = \frac{1}{2} \boldsymbol{\sigma} \cdot \mathbf{S}(\mathbf{r}, \mathbf{p}), \quad (9)$$

where $\boldsymbol{\sigma}$ is a four-dimensional vector of Pauli matrices

$$\begin{aligned} \sigma_0 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \\ \sigma_2 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \end{aligned} \quad (10)$$

The spatial Stokes parameters (2) can be derived from their spatial–angular counterparts (4) by removing the angular dependence

$$\mathbf{s}(\mathbf{r}) = \int d^2\mathbf{p} \mathbf{S}(\mathbf{r}, \mathbf{p}). \quad (11)$$

An scalar Wigner function for classical light waves including polarization can be defined in the form [8,9]

$$W(\mathbf{r}, \mathbf{p}, \boldsymbol{\Omega}) = \mathbf{S}(\mathbf{r}, \mathbf{p}) \cdot \boldsymbol{\Omega}, \quad (12)$$

where the four-dimensional vector $\boldsymbol{\Omega}$ is defined as

$$\boldsymbol{\Omega} = \frac{1}{2} \begin{pmatrix} 1 \\ \sqrt{3} \sin \theta \cos \phi \\ \sqrt{3} \sin \theta \sin \phi \\ \sqrt{3} \cos \theta \end{pmatrix}. \quad (13)$$

Since $\int d\Omega \Omega_j \Omega_k = \pi \delta_{j,k}$ the relation (12) can be inverted so that one can get $\mathbf{S}(\mathbf{r}, \mathbf{p})$ from $W(\mathbf{r}, \mathbf{p}, \boldsymbol{\Omega})$ in the form

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