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Entanglement and teleportation through thermal equilibrium state of spins in the *XXZ* model

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Abstract

We study the pairwise entanglement of a three-qubit spins in the *XXZ* model, and teleport an unknown state using the spin chain in the thermal equilibrium as a quantum channel. The effects of coupling strength, magnetic field, the anisotropy and temperature on the entanglement and fidelity are investigated. We find that the ferromagnetic spin chain is suitable for quantum teleportation, while the antiferromagnetic one is not. We give the maximal average fidelity, and the condition under which the maximal average fidelity is obtained. In addition, the relation between the entanglement and fidelity is studied, and we find that the considered entanglement cannot completely reflect the fidelity.

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Keywords: Spin chain; Entanglement; Teleportation; Concurrence; Fidelity

1. Introduction

Quantum entanglement is a fundamental concept in the quantum theory, and has been regarded as an essential physical resource for quantum computation and quantum communication [1]. An entangled system has nonlocal correlation between its subsystems. This nonlocal correlation makes some tasks possible or better than those done in classical world. For example, the nonlocal correlation allows one to teleport an unknown quantum state from one site to another site with fidelity better than any classical communication protocol [1].

Recently, quantum spin chains in condensed matter physics have been extensively studied in the context of quantum information science. On the one hand, they are very promising candidates for quantum information processing. In the spin chain systems, an unknown state, which is placed on one site, can be transmitted to a distant site with some fidelity by using the dynamics of the spin system [2]. The spin chains are also candidates for quantum information storing and quantum memories [3], quantum computation [4], quantum clone [5], teleportation [6,7], and so on. On the other hand, the entanglement in spin chains at finite temperatures has been investigated by many authors. Roughly those studies can be separated into at least three categories: the first one is about few-, mostly two-, spin chains [8–13]; the second concentrates on the chains of infinite spins with attention to quantum phase transitions [14–18]; the third investigates the chains of many but finite spins with the focus on the relation between the entanglement length and the correlation length [19–21].

In this paper, we also focus on the first category. Here we study the thermal pairwise entanglement (TPE) in 1D spin chain described by the *XXZ* model in an external magnetic field, and use the thermal equilibrium state of the chain as a quantum channel to teleport an unknown quantum state. Generally, at finite temperature, the state of the spin chain is a mixed entangled state. In the original

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protocol of quantum teleportation, the quantum channel is a pure maximally entangled state, e.g., an EPR (Einstein–Podolsky–Rosen) state [22]. However the decoherence from environment always impacts on the degree of entanglement, and the resource of pure entangled states is hard to prepare in a real experiment. Therefore it is necessary to consider the mixed state used to teleport a state, and the related work has been reported recently [23,24]. In this paper we use a concrete physical system, spin chain in thermal equilibrium state, to teleport any unknown one-qubit pure state, and the scheme of teleportation is given. We obtain the analytical and numerical results for the average fidelity at zero temperature and nonzero temperature, respectively. The effects of external magnetic field, coupling strength, anisotropy and temperature on the fidelity are investigated. In addition, the relation between the entanglement and fidelity is discussed.

The structure of the paper is as follows. In the second part we introduce the model in brief. In the third and fourth parts we discuss the TPE and teleportation, respectively. The paper ends in the fifths part with the conclusions.

2. The system and its thermal equilibrium state

We consider the XXZ model for a chain of three spins in an external magnetic field and with periodic boundary conditions. The corresponding Hamiltonian is given by

$$H = \sum_{i=1}^{3} (J\vec{\sigma}_i \otimes \vec{\sigma}_{i+1} + \Delta\sigma_i^z \otimes \sigma_{i+1}^z + B\sigma_i^z), \tag{1}$$

where J is the coupling strength in the x and y directions, and $J + \Delta$ is the coupling strength in the z direction. The parameter Δ quantifies the anisotropy in the interaction. The chain is said to be anti-ferromagnetic for $J > 0$ and ferromagnetic for $J < 0$. When $\Delta = -J$, the system reduces to XX model, and when $\Delta = 0$, it is XXX model. In Eq. (1) the term proportional to Δ commutes with the terms proportional to J, so the eigenstates of the Hamiltonian H coincide with those of the isotropic Heisenberg model. Simple calculation gives the eigenstates and the corresponding eigenvalues,

$$\begin{aligned}
|\psi_1\rangle &= \frac{1}{\sqrt{3}}(|110\rangle + |101\rangle + |011\rangle), \quad E_1 = 3J - B - \Delta,\\
|\psi_2\rangle &= \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle), \quad E_2 = 3J + B - \Delta,\\
|\psi_3\rangle &= \frac{1}{\sqrt{3}}(|110\rangle + p|101\rangle + p^2|011\rangle), \quad E_3 = -3J - B - \Delta,\\
|\psi_4\rangle &= \frac{1}{\sqrt{3}}(|110\rangle + p^2|101\rangle + p|011\rangle), \quad E_4 = -3J - B - \Delta,\\
|\psi_5\rangle &= \frac{1}{\sqrt{3}}(|001\rangle + p|010\rangle + p^2|100\rangle), \quad E_5 = -3J + B - \Delta,\\
|\psi_6\rangle &= \frac{1}{\sqrt{3}}(|001\rangle + p^2|010\rangle + p|100\rangle), \quad E_6 = -3J + B - \Delta,\\
|\psi_7\rangle &= |111\rangle, \quad E_7 = 3J - 3B + 3\Delta,\\
|\psi_8\rangle &= |000\rangle, \quad E_8 = 3J + 3B + 3\Delta,
\end{aligned} \tag{2}$$

where $p = \exp(2i\pi/3)$, $|0\rangle$ and $|1\rangle$ represent spin-up and spin-down states, respectively. The composite state in equilibrium at temperature T is characterized by a density matrix $\rho = \exp(-\beta H)/Z$, where $\beta = 1/(kT)$, k_B is the Boltzmann constant, and $Z = \mathrm{Tr}[\exp(-\beta H)]$ is the partition function. We expand the density operator of the system at a temperature T, which is given by its Gibbs states, as follows

$$\rho_{ABC} = \frac{\sum_{i=1}^{8} \exp(-\beta E_i)|\psi_i\rangle\langle\psi_i|}{\mathbf{Tr}\left(\sum_{i=1}^{8} \exp(-\beta E_i)|\psi_i\rangle\langle\psi_i|\right)}. \tag{3}$$

In order to study the entanglement between two spins, we require their reduced density matrices. The density matrix for different pair is obtained by tracing over the third spin, that is, $\rho_{AB} = \mathbf{Tr}_C\rho_{ABC}$, $\rho_{BC} = \mathbf{Tr}_A\rho_{ABC}$, and $\rho_{AC} = \mathbf{Tr}_B\rho_{ABC}$. By symmetry under cyclic shift, the three reduced density matrices are equal, and are labeled as ρ. In the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, ρ is given by

$$\rho = \frac{1}{3Z}\begin{pmatrix} v & 0 & 0 & 0 \\ 0 & w & z & 0 \\ 0 & z & w & 0 \\ 0 & 0 & 0 & \mu \end{pmatrix}, \tag{4}$$

where

$$\begin{aligned}
Z &= 2e^{-3\beta(J+\Delta)}\cosh 3\beta B + 2e^{\beta\Delta}[2\cosh 3\beta J + e^{3\beta J}]\cosh \beta B,\\
v &= 3e^{-3\beta(J+B+\Delta)} + e^{-\beta(3J-\Delta+B)} + 2e^{\beta(3J+\Delta-B)},\\
\omega &= 2e^{\beta\Delta}(e^{-3\beta J} + 2e^{3\beta J})\cosh \beta B,\\
z &= 2e^{\beta\Delta}(e^{-3\beta J} - e^{3\beta J})\cosh \beta B,\\
\mu &= e^{\beta(-3J+\Delta+B)} + 2e^{\beta(3J+B+\Delta)} + 3e^{3\beta(-J+B-\Delta)}.
\end{aligned} \tag{5}$$

In the next part we will use the above results to study the entanglement of two spins.

3. Thermal pairwise entanglement of the chain

In this part we first give the expression of the concurrence that quantifies the amount of entanglement between two qubits, then we study the entanglement between two spins when the temperature is zero and positive, respectively.

Convenient measure of entanglement of two-qubit state $\varrho$is the concurrence, which is defined as [25]

$$C(\varrho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{6}$$

where λ_i's are the square roots of the eigenvalues of the operator $R = \varrho\sigma^y \otimes \sigma^y\varrho^*\sigma^y \otimes \sigma^y$ in decreasing order, and the asterisk stands for the complex conjugate. For the state ρ given in Eq. (4), the four eigenvalues of the operator $\rho\sigma^y \otimes \sigma^y\rho^*\sigma^y \otimes \sigma^y$ are given by x_i ($i = 1, \ldots, 4$),