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Quantum phase-space description of light polarization

A.B. Klimov [a], J. Delgado [b], L.L. Sánchez-Soto [b,*]

[a] Departamento de Física, Universidad de Guadalajara, Revolución 1500, 44420 Guadalajara, Jalisco, Mexico
[b] Departamento de Óptica, Facultad de Física, Universidad Complutense, 28040 Madrid, Spain

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Abstract

We present a method to characterize the polarization state of a light field in the continuous-variable regime. Instead of using the abstract formalism of SU(2) quasidistributions, we model polarization as the superposition of two harmonic oscillators of the same angular frequency along two orthogonal axes, much in the classical way of dealing with this variable. By describing each oscillator by an s-parametrized quasidistribution, we derive in a consistent way the final function for the polarization. We compare with previous approaches and show how this formalism works in some relevant examples.

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1. Introduction

Polarization is a fundamental property of light, both in the quantum and in the classical domain. Although in quantum optics polarization has been mainly examined in the single-photon regime [1–7], different schemes have been proposed [8,9] and experimentally implemented [10,11] to characterize the continuous-variable limit of the quantum Stokes parameters. We stress that these continuous-variable polarization states can be carried by a bright laser beam, providing high bandwidth capabilities and therefore faster signal transfer rates than single-photon systems. In addition, they retain the advantage of not requiring the universal local oscillator necessary for other proposed continuous-variable quantum networks.

Since Wigner's seminal paper [12], and the remarkable contributions of Moyal [13], Stratonovich [14] and Berezin [15], it seems indisputable that phase-space methods, based on using quasidistributions that reflect the noncommutatibility

* Corresponding author. Tel.: +34 91 3944680; fax: +34 91 3944683.
E-mail address: lsanchez@fis.ucm.es (L.L. Sánchez-Soto).

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of quantum observables, constitute a valuable tool in examining continuous variables in quantum optics [16–18].

In particular, these methods have had great success in analyzing one-mode fields; i.e., Heisenberg–Weyl quasidistributions representing the quantum dynamics in the flat q–p (or, equivalently, a–a^*) space. Although not so popular in the quantum-optics community, spinlike systems, with the sphere $\mathscr{S}_2$ as phase space, have been discussed at length in this framework [19–27]. The resulting functions, naturally related to the SU(2) dynamical group, have been used to visualize, e.g., nonclassical properties of a collection of two-level atoms [28].

Since the Stokes operators can be formally identified with an angular momentum [29–32], one may naively expect a direct translation of these SU(2) quasidistributions to the problem of polarization. However, this is not the case, mainly because they act on different types of Hilbert spaces [33]. We can then conclude that the problem of an adequate quasiclassical description of polarization of light is still an open question [34,35].

The Stokes operators are a particular case of the Schwinger map [36], with two kinematically independent oscillators. In this spirit, it has been recently shown that the Stokes operators are the constants of motion of the two-dimensional isotropic harmonic oscillator [37]. This reflects the fact that the polarization of a classical field can be adequately viewed as the Lissajous figure traced out by the end of the electric vector of a monochromatic field [38]. In sharp contrast, in quantum optics the probability distribution for the electric field can be very far from having an elliptical form [39,40].

We wish to investigate this point from the perspective of quasidistributions. We show that one can start from the s-ordered quasidistributions from two kinematically independent oscillators: by eliminating an unessential common phase, we get well-behaved quasidistributions on the Poincaré sphere. We note in passing that this is the way in which polarization distributions are obtained in classical optics [41]. We apply the resulting family of polarization quasidistributions to some relevant states, and conclude that they constitute an appropriate tool to deal with such a basic variable.

2. Phase-space representation of a harmonic oscillator

To keep the discussion as self-contained as possible, we first briefly summarize the essential ingredients of phase-space functions for a harmonic oscillator that we shall need for our purposes.

In the Hilbert space $\mathscr{H}$, the state of the system is fully represented by its density operator $\hat{\varrho}$. In the phase-space formalism, $\hat{\varrho}$ is mapped by a family of functions (quasidistributions) $W^{(s)}(\alpha)$ onto the classical phase space X ($\alpha \in X$). This map is usually implemented by the generalized Weyl rule [26]

$$W^{(s)}(\alpha) = \mathrm{Tr}[\hat{\varrho}\hat{w}^{(s)}(\alpha)], \tag{1}$$

where the generating kernel $\hat{w}^{(s)}(\alpha)$ fulfill the properties

$$\hat{w}^{(s)}(\alpha) = [\hat{w}^{(s)}(\alpha)]^{\dagger}, \quad \int_X \mathrm{d}\mu(\alpha)\hat{w}^{(s)}(\alpha) = \hat{\mathbb{1}}. \tag{2}$$

The index s that labels functions in the family is related to the s ordering. The values $+1$, -1, and 0 correspond to the normal, antinormal, and symmetric ordering, respectively, or equivalently to the P, Q, and W functions. We stress that these quasidistributions can be determined in practice by using simple and efficient experimental procedures [42–47]. Moreover, they provide a simple measure of the nonclassical behavior of quantum states [48–51].

Let us now turn to the outstanding case of a harmonic oscillator described by annihilation and creation operators $\hat{a}$ and $\hat{a}^{\dagger}$, which obey the canonical commutation relation

$$[\hat{a}, \hat{a}^{\dagger}] = \hat{\mathbb{1}}. \tag{3}$$

The phase space is the complex plane $\mathbb{C}$ and the invariant measure is $\mathrm{d}\mu(\alpha) = \mathrm{d}^2\alpha/\pi$. The operator

$$\hat{D}(\alpha) = \exp(\alpha\hat{a}^{\dagger} - \alpha^*\hat{a}) \tag{4}$$

is the standard displacement operator in the complex plane α and leads to introduce the standard coherent states as