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Bose–Einstein solitons in time-dependent linear potential

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Abstract

In this paper, Bose–Einstein soliton solutions of the nonlinear Schrödinger equation with time-dependent linear potential are considered. Based on the F-expansion method, we present a number of Jacobian elliptic function solutions. Particular cases of these solutions, where the elliptic function modulus equals 1 and 0, are various localized solutions and trigonometric functions, respectively. Specially, for $V_{\text{ext}} = ZF(T) = Z[mg + H\cos(\omega_1 T)]$, we discussed the Bose–Einstein condensate trapped in the coupling external field with considering the effect of gravity; for $F(T) = \text{constant}$, it describes the wave (Langmuir or electromagnetic) in a linearly inhomogeneous plasma with cubic nonlinearly.

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Keywords: Bose–Einstein condensate; F-expansion method; Nonlinear Schrödinger equation; Solitons

1. Introduction

The experimental realization of Bose–Einstein condensation (BEC) in ultracold atomic gases [1,2] has triggered the experimental and theoretical exploration of the properties of Bose gases [3,4]. One of the important aspects in this area is the exploration of nonlinear properties of matter waves. Nonlinear excitations, such as solitons and vortices, have been observed in Bose–Einstein condensates (BEC's) [5–7] and the four-wave mixing has also been recently realized [8]. These studies have stimulated a large amount of research activities on nonlinear atom optics and other areas of condensed-matter physics and fluid dynamics [9–18].

The dynamic behavior of interacting bosonic gases at zero temperature is well described by the time-dependent Gross–Pitaevskii (GP) equation for the order parameter [4],

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$$i\hbar\frac{\partial\Psi}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2+V_{\text{ext}}+g|\Psi|^2\right]\Psi, \tag{1}$$

where $\int d\vec{r}|\Psi|^2=N$ is the number of atoms in the condensate, $g=4\pi\hbar^2a/m$ is the interacting constant with m the mass of the atom, and a the s-wave scattering length ($a>0$ for repulsive interaction; while $a<0$ for attractive interaction). V_{ext} is the external potential. In fact, besides internal interactions, the macroscopic behavior of BEC matter is highly sensitive to external conditions, and primarily to the external trapping potential. The external potential has many forms, such as harmonic oscillator potential, optical lattice potential [19,20], elliptic function potential [21–23], double well potential [24–27], and so on.

As mentioned in [28], the trapped quasi-low-dimensional condensates will offer many possibilities for investigating the nonlinear excitations such as solitons and vortices, which are more stable than in 3D, where the solitons suffer from the transverse instability and the vortices can bend [29–32]. Thus both theory and experiment call for a detailed study on the soliton excitations in quasi-low-dimensional BECs. When the transverse dimensions of the condensate are on the order of its healing length and its longitudinal dimension is much longer than its transverse ones, the GP equation reduced to the quasi-one-dimensional (quasi-1D) regime of the GP equation. So in this paper, we discussed the quasi-one-dimensional BEC trapped in the time-dependent linear potential.

It is worth noting that the 1D nonlinear Schrödinger equation (NLSE) in a time-dependent external potential has been discussed in some papers [33,34]. They employed Husimi's transformation [35] to change the space variable from x to x' where $x'=x-\xi(t)$. This x' is the coordinate with respect to the moving origin $\xi(t)$. Later $\xi(t)$ will be taken as the center of mass of the soliton. Finally, they obtain the one-soliton of NLSE with a time-dependent linear potential. However, in our paper, based on the F-expansion method, we obtained the soliton solutions much more easily and directly than [33,34]. Moreover, we present a number of Jacobian elliptic function solutions of BEC in the time-dependent linear potential. Particular cases of these solutions, where the elliptic function modulus equals 1 and 0, are various localized solutions and trigonometric function solutions, respectively.

2. Bose–Einstein soliton of the time-dependent linear potential

If considering the mean-field model of a quasi-1D BEC trapped in the time-dependent linear potential which is given by the following nonlinear Schrödinger equation:

$$i\hbar\frac{\partial\Phi(Z,T)}{\partial T}=-\frac{\hbar^2}{2m}\frac{\partial^2\Phi(Z,T)}{\partial Z^2}+ZF(T)\Phi(Z,T)+\frac{4\pi\hbar^2a}{m}|\Phi(Z,T)|^2\Phi(Z,T), \tag{2}$$

where $\Phi(Z,T)$ is the macroscopic wave function of the condensate, $F(T)$ denotes the arbitrary function of time T.

In order to simplify the Eq. (2), we introduce the dimensionless variables $t=\omega_0T, z=Z\sqrt{m\omega_0/\hbar}$, and $\psi=\Phi\sqrt{\hbar/Nm\omega_0}$, then we obtain the following dimensionless equation:

$$i\frac{\partial\psi}{\partial t}+\frac{1}{2}\frac{\partial^2\psi}{\partial z^2}+\eta|\psi|^2\psi+zf(t)\psi=0, \tag{3}$$

where $\eta=-4\pi aN$, $f(t)=-\frac{\sqrt{m\omega_0\hbar}}{m\omega_0^2\hbar}F(\frac{t}{\omega_0})$. When the s-wave scattering length $a>0$, i.e., $\eta<0$, the interaction between the particles in the condensate is repulsive. In opposite case $a<0$, i.e., $\eta>0$, the interaction is attractive.

In order to obtain other new quasi-soliton solutions of Eq. (3), we introduce the following auxiliary equation:

$$\phi'^2=c_0+c_2\phi^2+c_4\phi^4, \tag{4}$$