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Mode expansions in the quantum electrodynamics of photonic media with disorder

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Abstract: We address two issues in the quantum electrodynamical description of photonic media with some disorder, neglecting material dispersion. When choosing a gauge in which the static potential vanishes, the normal modes of the medium with disorder satisfy a different transversality condition than the modes of the ideal medium. Our first result is an integral equation for optical modes such that all perturbation-theory solutions by construction satisfy the desired transversality condition. Secondly, when expanding the vector potential for the medium with disorder in terms of the normal modes of the ideal structure, we find the gauge transformation that conveniently makes the static potential zero, thereby generalizing work by Glauber and Lewenstein (1991) [15]. Our results are relevant for the quantum optics of disordered photonic crystals.

Keywords: Field quantization; Photonic crystals; Disorder; Mode expansions; Gauge transformations

1. Introduction

The quantum optics of random media is a young research field, studying the effects of randomness on quantum correlations and entanglement of quantum states of light in a multi-mode setting [1–6]. Traditionally, random media are studied with randomness against a homogeneous dielectric background. Recently researchers also realized that every real photonic structure, such as a photonic crystal, is in a sense a random medium, since there is inevitably some randomness on top of the ideal dielectric properties [7–9]. The interplay between the randomness and an ordered inhomogeneous dielectric background can sometimes be exploited. For example, in a photonic-crystal background slow light can promote localization of light due to even minute random scattering [10–13].

The typical starting point in the quantum optics of random media is the assumption of a multimode scattering matrix with elements subject to disorder [1,2,4,5]. One level of modeling deeper is the quantum electrodynamics (QED) of these media, to derive the form of the scattering matrix and its dependence on the types of disorder in the medium. Here we aim to contribute to this QED description for spatially inhomogeneous media with some additional disorder. For simplicity we assume that material dispersion can be neglected, as in Refs. [14–18], although more general quantized-field theories for dispersive and absorbing inhomogeneous dielectric media have also been developed [19–22].

First we will derive a useful new integral equation for the normal optical modes in a photonic medium with disorder. Related work on integral equations and Green-function methods can be found in Refs. [23–32], and on disorder in photonic media in Refs. [33–42]. Instead of an integral equation for the modes involving a disorder potential and the usual Green tensor $\mathbf{G}$ of the

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unperturbed medium, we introduce an alternative integral equation involving a kernel **K** [27] that differs from **G** (details below). This **K** emerged naturally in a quantum optical description of light sources and scatterers in a photonic environment [27], and has since then been frequently employed in a quantum optics context, e.g. in Refs. [43–46]. Here instead we propose a novel use to it, in an integral equation that we derive for optical modes of photonic media with disorder. We discuss its specific advantage that arbitrary-order perturbation-theory solutions automatically satisfy a desirable gauge condition.

As our second topic we discuss an alternative to a normal-mode expansion, namely an expansion into modes that get coupled because of a perturbation. Disorder is such a perturbation. Several methods have been developed to describe disorder in photonic crystals [33,34,37,39–42]. These methods are useful both in the classical and in the quantum optics of disordered photonic crystals. Here our aim will be to generalize the coupled plane-wave description by Glauber and Lewenstein [15], to a coupled Bloch-mode description for example. Although there will be coupled modes in our theory, it is different from what is commonly known as a 'coupled-mode theory' [47,13], which also finds application in quantum optics [48].

The structure of this article is as follows: in Section 2 we briefly review the quantum electrodynamics of inhomogeneous dielectric media. In Section 3 a useful integral equation is derived for the independent optical modes, which is especially well suited for perturbation theory calculations of the effects of disorder on normal modes. Section 4 defines the problem to find a convenient gauge transformation when starting from an expansion into modes that are not the normal modes. This transformation is constructed in Section 5, specified to the special case of a plane-wave expansion in Section 6, before we conclude in Section 7.

2. Normal-mode expansion

The quantum optical description of the electromagnetic field in a photonic medium with negligible material dispersion starts with the source-free Maxwell equations in matter $\nabla\cdot\mathbf{D}=0$, $\nabla\cdot\mathbf{B}=0$, $\dot{\mathbf{D}}=\nabla\times\mathbf{H}$, and $\dot{\mathbf{B}}=-\nabla\times\mathbf{E}$ and the constitutive relations for a lossless nonmagnetic medium, $\mathbf{D}(\mathbf{r})=\varepsilon_0\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r})$ and $\mathbf{B}(\mathbf{r})=\mu_0\mathbf{H}(\mathbf{r})$. We can introduce a vector potential $\mathbf{A}(\mathbf{r})$ and a scalar potential $\Phi(\mathbf{r})$ such that

$$\mathbf{E}=-\nabla\Phi-\dot{\mathbf{A}}, \tag{1}$$

$$\mathbf{B}=\nabla\times\mathbf{A}, \tag{2}$$

and where the dot denotes a time derivative. There is gauge freedom, *i.e.* one can make combined changes of the vector potential $\mathbf{A}\to\mathbf{A}+\nabla\chi$ and the static potential $\Phi\to\Phi-\dot{\chi}$ that leave the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ unaltered. Here $\chi(\mathbf{r},t)$ is an arbitrary scalar function of space and time.

The usual steps from classical to quantum electrodynamics of photonic media are first to choose a convenient gauge, then to identify the canonical fields, next to express those fields into normal modes, and finally to associate non-commuting operators with them [15,16,18]. In the first step, the simplest gauge choice for lossless nondispersive photonic media is a generalization of the free-space Coulomb gauge $\nabla\cdot\mathbf{A}=0$, namely the 'generalized Coulomb gauge'

$$\nabla\cdot[\varepsilon(\mathbf{r})\mathbf{A}]=0. \tag{3}$$

We call such $\mathbf{A}$ 'generalized transverse' or 'ε-transverse', while Ref. [49] uses 'quasi-transverse'. Advantage of this gauge choice is that the corresponding static potential Φ can be chosen identically zero, which leaves the vector potential as the only canonical variable in the Lagrangian density $\mathcal{L}$ that leads to Maxwell's equations,

$$\mathcal{L}=\frac{1}{2}\varepsilon_0\varepsilon(\mathbf{r})\left[\frac{\dot{\mathbf{A}}(\mathbf{r},t)}{c}\right]^2-\mu_0^{-1}[\nabla\times\mathbf{A}(\mathbf{r},t)]^2. \tag{4}$$

The field canonically conjugate to the vector potential equals $\varepsilon_0\varepsilon(\mathbf{r})\dot{\mathbf{A}}(\mathbf{r})=-\mathbf{D}(\mathbf{r})$, and the Hamiltonian density becomes

$$\mathcal{H}=\frac{[\mathbf{D}(\mathbf{r})]^2}{2\varepsilon_0\varepsilon(\mathbf{r})}+\frac{1}{2\mu_0}[\nabla\times\mathbf{A}(\mathbf{r})]^2. \tag{5}$$

One can introduce commutation relations for $\mathbf{A}$ and $-\mathbf{D}$ directly, but a simpler equivalent procedure is to expand the fields into normal modes. The wave equation for the vector potential is

$$\nabla\times\nabla\times\mathbf{A}+\frac{\varepsilon(\mathbf{r})}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}=0. \tag{6}$$

In terms of the complete set of normal modes $\mathbf{f}_\lambda(\mathbf{r})$ with mode index λ that satisfy

$$-\nabla\times\nabla\times\mathbf{f}_\lambda(\mathbf{r})+\varepsilon(\mathbf{r})\frac{\omega_\lambda^2}{c^2}\mathbf{f}_\lambda(\mathbf{r})=0, \tag{7}$$

the vector potential can be expressed as

$$\mathbf{A}(\mathbf{r},t)=\sum_\lambda\sqrt{\frac{\hbar}{2\varepsilon_0\omega_\lambda}}\left[\hat{a}_\lambda(t)\mathbf{f}_\lambda(\mathbf{r})+\hat{a}_\lambda^\dagger(t)\mathbf{f}_\lambda^*(\mathbf{r})\right]. \tag{8}$$