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Comparison of spatial harmonics in infinite and finite Bragg stacks for metamaterial homogenization☆

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Abstract

Metamaterial homogenization may be based on the dominance of a single Floquet–Bloch spatial harmonic in an infinite periodic structure – with the dominance quantified in terms of the relative magnitude of the associated spatial harmonic Poynting vector. For the corresponding finite structure the field is not quasi-periodic and cannot be expanded in Floquet–Bloch spatial harmonics; however, a set of pseudo spatial harmonics can be defined and the dominance of a single such harmonic likewise be used to determine whether the structure can be homogenized. For three different lossless Bragg stack configurations (one of which is magneto-dielectric), we show, using spectral representation, that the field in the finite structure can be accurately expanded in terms of these pseudo spatial harmonics and that the distribution of these agrees very well with the distribution of Floquet–Bloch spatial harmonics of the corresponding infinite Bragg stack. This is even the case for finite Bragg stacks having only two unit cells; thus, the number of unit cells does not influence the homogenizability of this type of configuration.

Keywords: Spatial harmonics; Floquet–Bloch; Homogenization; Metamaterials; Bragg stack; Truncation

1. Introduction

For more than a decade, metamaterials (MMs) have received an immense interest from both the electromagnetics and photonics research communities [1–3]. MMs are artificially structured materials composed, in general, of periodic arrangements of identical unit cells that can assume a wide variety of configurations and are therefore inherently inhomogeneous. However, homogeneous MMs are suited to realize concepts such as the perfect lens [4] and cloaking devices [5].

Homogenization is a key element in metamaterial science and, in extension of classical mixing rules for composite materials [6], the past 15 years have witnessed the development of several widely different techniques for metamaterial homogenization [7–17] which nonetheless continues to be of strong research interest [18,19]. A thorough discussion of these several homogenization techniques, and their many variations, is outside the scope of the present work where we employ Floquet–Bloch analysis to compare homogenization of infinite and finite periodic metamaterial structures.

For infinite periodic structures, homogenization can be based on the Floquet–Bloch spatial harmonics [20,21,17,22,23]. However, for finite periodic structures, the field is not quasi-periodic (periodic except for a phase change) and it cannot be expanded exactly in

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the discrete Floquet–Bloch spatial harmonics. This is an important distinction between infinite and finite periodic structures. For finite structures, approaches such as inverting the scattering parameters [24,25] referred to as the Nicolson–Ross–Weir (NRW) method, has proven useful for many practical purposes [7], and the ambiguity of this technique is now well understood [26]. However, this method does not provide the same insight to the homogenizability of the material as the Floquet–Bloch spatial harmonics do. In other words, the NRW method yields the equivalent material parameters for a given set of scattering parameters, while not taking into account the actual fields inside the periodic structure.

The purpose of this work is to extract and quantify the dominance of pseudo spatial harmonics of the fields in finite Bragg stacks (FBSs). These are compared to the Floquet–Bloch spatial harmonics of the fields in infinite Bragg stacks (IBSs). For a finite periodic structure (such as the FBS), the spatial spectrum is continuous and thus it does not strictly make sense to talk about spatial harmonics, as these are inherently discrete, hence the prefix *pseudo*. The spatial spectrum of an infinite periodic structure (such as the IBS), however, consists of exactly such discrete harmonics. Using the spatial harmonic Poynting vectors (the Poynting vector of the individual harmonic) of the Floquet–Bloch spatial harmonics and the pseudo spatial harmonics, we can quantify the dominance of each harmonic. This also enables us to compare structures in terms of number of unit cells to observe the effect of structural truncation. It is noted that the dominance of the spatial harmonic Poynting vector is not the only possible criteria for homogenization; but it has several advantages. First, it takes into account both electric and magnetic fields. Second, the sum of the Poynting vectors of all spatial harmonics equal the power flow in the infinite structure. Third, when a single harmonic is dominating – and the structure thus can be homogenized – its Poynting vector closely approximates the power flow of the total field.

The focus of the present work is on the expansion of fields in finite structures in terms of pseudo spatial harmonics, and the potential dominance of an individual harmonic, which is important for homogenization; but we do not retrieve any material parameters for the harmonics as was done in, e.g. [22]. Our investigations are focused on dielectric and magneto-dielectric lossless 1D Bragg stacks. These facilitate exact analytical solutions for the total fields in the finite as well as infinite structures; it thus avoids the approximations necessary for most other types of unit cells and periodic structures.

This manuscript is organized as follows: In Section 2, the Bragg stack configuration is described. Section 3 is divided into three parts. First, the Floquet–Bloch analysis is applied to the IBS, and both the fields and the spatial harmonics are accurately described. Second, the analysis of a FBS is carried out in a manner analogous to that of the IBS. The exact field of the FBS is expanded approximately in a set of pseudo spatial harmonics, the Fourier transform of which are continuous sinc functions. Lastly we develop an expression for the spatial harmonic Poynting vectors; this is eventually used to assess the dominance of each spatial harmonic. In Section 4 the implementation and numerical techniques are described. Subsequently, in Section 5 we show our analytical and numerical results. We show the dispersion diagram of the IBSs, the continuous field spectra for the FBSs, and lastly compare the pseudo spatial harmonics of the FBSs with the Floquet–Bloch spatial harmonics of the corresponding IBSs. This is done for three different unit cell layouts, and for different number of unit cells in the FBSs. The entire work is summarized and concluded upon in Section 6.

Throughout this work the time factor $e^{j\omega t}$ is assumed and suppressed, where ω is the angular frequency, t is time and j is the imaginary unit.

After initial submission of this work, [27] appeared in ArXiv in March 2014. In the reference, fields in finite 1D periodic structures are also investigated in analytical terms, and an explicit mathematical relation between fields in FBSs and Floquet–Bloch harmonics is established.

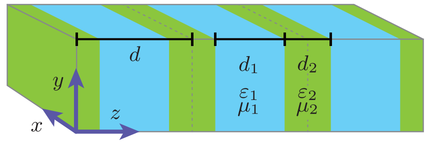

Fig. 1. The bi-layered Bragg stack. The layers are planar slabs infinite in the x and y directions.

2. Configuration

The configuration of the 1D bi-layered Bragg stack is shown in Fig. 1.

Its unit cell consists of two materials, the center planar slab surrounded by two planar slabs of equal thickness, in order to make the unit cell symmetric [28].

The permittivity and permeability of the center slab are denoted by ε_1 and μ_1, respectively, whereas those of the surrounding two slabs are denoted by ε_2 and μ_2, respectively. Both materials are lossless and non-dispersive. The thickness of the center slab is d_1 and the combined thickness of the surrounding slabs is d_2. The total thickness of the unit cell is $d = d_1 + d_2$.