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Phase space analysis of metamaterial-based optical systems

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Abstract

Phase space analysis of light refraction in optical systems consisting of slabs or thin lenses from either metamaterials with negative refractive indices or common materials is performed with the aim of finding the conditions of perfect imaging for metamaterial-based optical systems. The analysis in the paraxial approximation uses ABCD matrices, whereas full ray tracing is employed in the non-paraxial case. The phase space analysis reveals that the ideality of planar metamaterial lenses only occurs when the absolute value of the refractive index in metamaterials is the same as in the surrounding medium.

Keywords: Metamaterials; Phase space

1. Introduction

Metamaterials are artificial materials with dimensions much smaller than the excitation wavelength which can, in certain conditions, have an effective negative index of refraction n. Throughout this paper we restrict the meaning of metamaterials to this particular case, of $n < 0$. Among their unusual properties, metamaterials can cloak optical signals and can be used to fabricate planar superlenses that overcome the common diffraction limit. There are already an impressive number of papers dealing with several aspects of light propagation through metamaterials, the majority of them revealing significant differences with respect to propagation through media with a positive refractive index. For recent reviews on metamaterials, see Refs. [1–7] and the references therein.

The aim of this paper is to explore another method to compare optical systems containing metamaterials with $n < 0$ with systems containing materials with $n > 0$. More precisely, we develop a classical phase space treatment for light propagation [8,9] through metamaterial-based slabs and thin lenses and compare the outcome with the known results for similar optical systems containing common materials. The paraxial approximation is first used, but light propagation in the nonparaxial case is later studied to investigate the performances of planar metamaterial lenses. In particular, it is found that these planar lenses are ideal with respect to spherical aberrations, for example, as long as the absolute value of the refractive indices of the metamaterial and the surrounding medium are the same. In all other cases spherical aberrations are present also in planar metamaterial lenses. Moreover, evanescent propagation through planar lenses is described in phase space, the amplification of these waves being modeled through non-unit determinant matrices. Throughout the

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paper we consider propagation of coherent light beams along the z direction chosen as optical axis and restrict our treatment to only one transverse coordinate, x, for reasons of graphical representation. In this case, the phase space is two-dimensional and spanned by the canonically conjugated variables, x and p. (In the general case, with two transverse coordinates, the phase space is four-dimensional and hence impossible to represent.) Light propagation through anisotropic media, especially through anisotropic metamaterials, can be accounted for either considering the complete four-dimensional phase space or treating separately, in a two-dimensional phase space, the propagation in the two transverse directions.

2. Light propagation in phase space

The phase space representation in optics is widely used in connection with the Hamiltonian formalism [10], in which the trajectory of a ray in the (x,z) plane through a medium with refractive index $n(x)$ is described by the equations

$$\frac{\partial x}{\partial z}=\frac{\partial H}{\partial p}, \quad \frac{\partial p}{\partial z}=-\frac{\partial H}{\partial x}, \tag{1}$$

where the geometrical optical Hamiltonian is

$$H(x,p)=-\sqrt{n^2(x)-p^2}. \tag{2}$$

In the paraxial approximation, when both x and the propagation angle θ (the angle between the ray and the z axis) are small enough, $p=n\theta$.

In geometrical optics, a ray with a ray vector $(x,p)^T$ (the superscript T indicates transposition) at a plane z = const. is represented in phase space by a point, and a light source is regarded as a bundle of independent rays with an associated closed area in phase space. According to the Liouville theorem [11], this area remains constant under canonical transformations. In particular, the evolution of a ray in the paraxial approximation between z = const. planes can be expressed in a matrix form:

$$\begin{pmatrix} x \\ p \end{pmatrix}_o=\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} x \\ p \end{pmatrix}_i=M\begin{pmatrix} x \\ p \end{pmatrix}_i, \tag{3}$$

where o and i label the value of parameters at output and input planes, respectively, and the unit-determinant matrix M is symplectic [11]. In the two-dimensional phase space considered here the matrix elements A, B, C, D are numbers. More precisely, for propagation along a distance d through a medium with refractive index n, $A = D = 1$, $C = 0$, $B = d/n$ and for a thin lens $A = D = 1$, $B = 0$, $C=-1/f$, the total matrix of an optical system being obtained by multiplying the matrices of each component of the system.

In wave optics, light beams can be represented in phase space by a number of distribution functions; we choose to represent them by means of the Wigner distribution function (WDF) [12,13] due to the property that its evolution through first-order (paraxial) optical systems is simply related to the elements of M [8]:

$$W_o(x,p)=W_i(Dx-Bp,-Cx+Ap). \tag{4}$$

The WDF of a scalar, coherent light source with a field distribution $\varphi(x)$ is defined as

$$W(x,p)=(2\pi)^{-1}\int\varphi\left(x+\frac{x'}{2}\right)\varphi^*\left(x-\frac{x'}{2}\right)\exp(ikpx')dx'. \tag{5}$$

where k is the wavenumber. The definition in (5) is chosen such that the p variable has the same dimensionality as in the ray matrix formalism in (3).

3. Light propagation through a slab in the paraxial approximation

Let us consider first an optical system composed of a slab with thickness d made from a material with refractive index n ($n < 0$ for metamaterials and $n > 0$ for common materials) in air (surrounded by media with $n = 1$). We assume that a light source is placed at $z = 0$, at a distance d_1 in front of the slab (see Fig. 1(a)) and trace its phase space evolution at several planes along the z axis. In particular, if the light source is a Gaussian beam [14] with waist w_0 at $z = 0$, the normalized scalar field distribution at the waist is

$$\varphi(x)=\exp\left(\frac{-x^2}{2w_0^2}\right), \tag{6}$$

and the corresponding WDF is given by

$$W(x,p)=\sqrt{2\pi}w_0\exp\left(\frac{-x^2}{w_0^2}-p^2k^2w_0^2\right). \tag{7}$$

For a clearer representation of light propagation, we represent throughout this paper the WDF through its contour at $1/e$ from its maximum height. These contours are displayed in Fig. 2(a) and (b) for the light distribution at the $z = 0$ plane (solid black line), for $z=d_1$ (in front of the slab, dashed black line), for $z=d_1+d$ (immediately after the slab, dashed gray line) and at a distance d_2 after the slab (solid gray line). The representation in Fig. 2(a) has been performed in normalized coordinates, $X=x/w_0$ and $P=kpw_0$, for