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Determining polarizability tensors for an arbitrary small electromagnetic scatterer

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Abstract

In this paper, we present a method to retrieve tensor polarizabilities of general bi-anisotropic particles from their far-field responses to plane-wave illuminations. The necessary number of probing excitations and the directions where the scattered fields need to be calculated or measured have been found. When implemented numerically, the method does not require any spherical harmonic expansion nor direct calculation of dipole moments, but only calculations of co- and cross-polarized scattering cross sections for a number of plane-wave excitations. With this simple approach, the polarizabilities can be found also from experimentally measured cross sections. The method is exemplified considering two bi-anisotropic particles, a reciprocal omega particle and a non-reciprocal particle containing a ferrite inclusion coupled to metal strips.

Keywords: Bi-anisotropic particle; Polarizability; Dipole moments; Scattered fields

1. Introduction

Artificial materials (metamaterials) made of small inclusions (meta-atoms) positioned beside each other have become very popular, because with this concept it is possible to realize exotic electromagnetic properties which are not found in natural materials. These inclusions, so-called meta-atoms, which are electrically small (in comparison to the wavelength in the medium), in the sense of their electromagnetic response play the same role as atoms do in natural materials. The averaged electric and magnetic properties of a metamaterial sample are determined by electric and magnetic properties of individual inclusions and by their mutual interactions. The meta-atoms can be characterized by their polarizabilities. The polarizabilities show how a single meta-atom behaves in responding to external electromagnetic fields. Knowing the electromagnetic properties of each building block of the metamaterial allows us to understand the electromagnetic properties of metamaterials as composite media. In particular, proper engineering of meta-atoms allows us to design metamaterials with required effective properties. There are several approaches to determination of effective parameters of a medium knowing the electric and magnetic moments of each

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building block of the medium, both for volumetric (bulk) samples and thin layers (metasurfaces), e.g. [1–6]. A method of calculating macroscopic parameters for reciprocal bianisotropic composites was proposed in [7]. In this paper, we discuss how the meta-atom polarizabilities can be retrieved from the knowledge of single meta-atom far-field scattering response to plane-wave excitations.

The spherical harmonic expansion theory introduced by Mie for a homogeneous sphere of any size and arbitrary refractive index is known as one of the main tools in deriving the polarizabilities of a single dielectric sphere [8–10]. Later, this theory was extended to particles of an arbitrary shape [11]. Recently, the extended theory has been used by many researchers to study the multipolar behavior of special inclusions, e.g. [12,13]. Using the generalized Mie theory and writing the scattered fields in terms of vector spherical harmonics, multipolar moments of an arbitrary scatterer can be calculated. However, this approach implies computationally heavy integrations of scattered fields over the sphere surrounding the particle that complicates the implementation of the method in numerical calculations. Furthermore, it appears problematic to use such methods for extracting polarizabilities from experimentally measured response of the particle.

In most cases when the particle is electrically small, electric and magnetic dipolar moments are the only significant and important moments in the Mie expansion. This assumption allows us to dramatically simplify the scattering-based polarizability retrieval and propose a much simpler method for extracting polarizability tensors of an arbitrary small scatterer from its response in the far zone. This technique for the polarizability extraction is based on the assumption that in the far zone the contributions to the scattered fields from electric and magnetic dipoles dominate over those from higher-order multipole moments of the particle. The validity of this assumption can be tested by numerical evaluation of the scattering pattern or, experimentally, by repeating the measurements with several orientations of the particle. Let us note that for the needs of designing metasurfaces and metamaterials the knowledge of higher-order polarizabilities is usually not required (the plane-wave reflection and transmission coefficients from regular metasurfaces as well as the effective parameters of metamaterials are defined by dipolar responses of the inclusions). However, in the present technique the particles are tested in the free-space environment where the multipole moments may contribute to the measured quantities, and only this leads to the limitation of the method applicability to particles with predominately dipolar response. This limitation can be lifted if the particle is tested in a closed waveguide environment, where the fundamental-mode reflected and transmitted fields depend only on the particle dipole moments.

To extract one specific polarizability component of the scatterer, our method implies determination of the scattered fields only in two special directions. This significantly simplifies the realization of the method in numerical calculations. Furthermore, the discrete and minimal number of directions in which the scattered fields must be probed allows us to utilize the method also experimentally. This method for the first time was proposed in [14] for helical particles possessing bi-anisotropic electromagnetic coupling. In the present paper, we generalize the polarizability retrieval method so that it can be utilized for arbitrary small particles with the most general bi-anisotropic properties. The method can be considered as a generalization of the approach used in [15] for determination of the polarizablities of small chiral particles from their co- and cross-polarized scattering cross sections.

In the most general case, assuming that the induced dipole moments in the particle depend linearly on the applied fields, the dipolar moments induced in the particle relate to the incident fields (at the location of the particle) by the polarizability tensors as:

$$\begin{aligned}\mathbf{p} &= \overline{\overline{\alpha}}_{\rm ee}\cdot\mathbf{E}_{\rm inc}+\overline{\overline{\alpha}}_{\rm em}\cdot\mathbf{H}_{\rm inc},\\ \mathbf{m} &= \overline{\overline{\alpha}}_{\rm me}\cdot\mathbf{E}_{\rm inc}+\overline{\overline{\alpha}}_{\rm mm}\cdot\mathbf{H}_{\rm inc}.\end{aligned} \tag{1}$$

These relations hold for bi-anisotropic particles of all known classes: reciprocal chiral and omega, non-reciprocal Tellegen and "moving" particles, and any combination of these [16,17]. For a special case of anisotropic particles without electromagnetic coupling (e.g., small dielectric spheres) the relations are simplified taking into account that $\overline{\overline{\alpha}}_{\rm em}=\overline{\overline{\alpha}}_{\rm me}=0$.

The structure of the paper is as follows. In Section 2, we formulate the basic idea and derive the proper expressions for the polarizabilities of a general bi-anisotropic particle (assuming the particle is electrically small). In Section 3, we implement the method for two different particles: a reciprocal omega particle and a non-reciprocal particle possessing "moving" and chiral electromagnetic couplings.

2. Basic formulation

2.1. Polarizabilities of a bi-anisotropic particle

In order to determine the polarizabilities of an arbitrary particle, we analyze the far-field response of