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PHOTONICS AND NANOSTRUCTURES
Fundamentals and Applications

Photonics and Nanostructures – Fundamentals and Applications 12 (2014) 312–318

www.elsevier.com/locate/photonics

Analogue transformation acoustics and the compression of spacetime

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Received 16 January 2014; received in revised form 9 April 2014; accepted 5 May 2014
Available online 13 May 2014

Abstract

A recently developed technique known as analogue transformation acoustics has allowed the extension of the transformational paradigm to general spacetime transformations under which the acoustic equations are not form invariant. In this paper, we review the fundamentals of analogue transformation acoustics and show how this technique can be applied to build a device that increases the density of events within a given spacetime region by simultaneously compressing space and time.

Keywords: Metamaterials; Transformation acoustics; Analogue gravity; Spacetime; Compressor

1. Introduction

Metamaterials offer an unprecedented flexibility in the construction of media with properties that are difficult or impossible to find in nature [1]. This concept first appeared within the frame of electromagnetism, and enabled scientists to design exotic devices such as negative-index superlenses [2]. Afterwards, the notion of metamaterial has been extended to other branches of physics [3], such as acoustics [4,5], electronics [6] or thermodynamics [7,8]. To take full advantage of this flexibility in the synthesis of tailor-made properties, we also need new design techniques that help us to engineer these properties with the aim of building novel devices with advanced functionalities. Along this line, one of the most powerful techniques is transformation optics, which prescribes the properties that a medium should have in order to alter the propagation of light in almost any imaginable way [9–12]. As a result, metamaterials and transformation optics have teamed up to open the door to the realization of photonic devices that were unthinkable only a few years ago, such as invisibility cloaks or optical wormholes [13,14], constituting one of the most interesting recent developments in material science. The great success of

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http://dx.doi.org/10.1016/j.photonics.2014.05.001
1569-4410/

the transformational paradigm in the field of electromagnetism has led the research community to look for ways in which this approach could be extended to other fields [15,16,7].

Noticing that the key to transformation optics is the form invariance of Maxwell's equations under any spacetime transformation, the initial approach was to try to exploit form invariance in the governing equations of different physical phenomena. Therefore, one of the crucial issues in transformational methods is the range of coordinate transformations over which the relevant field equations have this property [17–20]. Outside of optics, acoustics is probably the field in which the greatest advance has been achieved. There, the form invariance of the acoustic equations under spatial transformations has been used to obtain the material parameters that deform acoustic space in the desired way, e.g., for cloaking acoustic waves [4,16,21–27].

However, this approach to transformation acoustics has been undermined by the deep structural differences between Maxwell's theory with its underlying relativistic geometry on the one hand, and the Galilean character of fluid mechanics on the other hand, which reduces the power of the traditional transformational method when applied to acoustics. Specifically, classical acoustic equations are not form invariant under transformations that mix space and time [20]. As a consequence, the method cannot be applied to design devices based on this kind of transformation, contrarily to what has been done in optics [28–30].

Recently, the problem of transformation acoustics was approached from another angle [31,20]. Instead of using directly the symmetries of the acoustic equations to bridge between different solutions for the propagation of acoustic waves, the symmetries of an analogue abstract spacetime (described by relativistic form-invariant equations) are exploited. In this method, each couple of solutions connected by a general coordinate transformation in the analogue spacetime can be mapped to acoustic space. This way, it is possible to find the relation between the acoustic material parameters associated with each of these transformation-connected solutions. The result is an alternative version of transformation acoustics as powerful as its optical counterpart and that we refer to as analogue transformations acoustics (ATA).

In this paper, we review the ATA method and some of the devices it has allowed us to engineer, which were unworkable through other approaches (Section 2). Subsequently, we use this technique to design a new kind of spacetime compressor that increases the density of events in a given region. The performance of the device is analyzed and verified through numerical calculations (Section 3). The differences between the proposed spacetime compressor and other squeezing devices are discussed. Finally, some conclusions are drawn (Section 4).

2. Analogue transformation acoustics

The extension of transformation acoustics to general spacetime transformations presents two separate problems. On the one hand, the acoustic equations are not form invariant under transformations that mix space and time. As mentioned above, this drawback can be circumvented with the aid of an auxiliary relativistic spacetime. On the other hand, it has been shown that the acoustic systems usually considered in transformation acoustics (which deal with the propagation of acoustic waves in stationary or non-moving fluids) do not posses enough degrees of freedom so as to mimic an arbitrary spacetime transformation. This is the case of the system represented by the standard pressure wave equation [20]. The limitations of transformational pressure acoustics have also been analyzed by other authors [32].

Instead of the pressure wave equation, ATA uses the wave equation for the velocity potential ϕ_1 (defined as $-\nabla\phi_1 = \mathbf{v}_1$, where $\mathbf{v}_1$ is the velocity of the acoustic perturbation), which reads

$$
\begin{aligned}
&-\partial_t(\rho_V {c_V}^{-2}(\partial_t\phi_1 + \mathbf{v}_V\cdot\nabla\phi_1)) + \nabla \\
&\quad \cdot(\rho_V\nabla\phi_1 - \rho_V c_V^{-2}(\partial_t\phi_1 + \mathbf{v}_V\cdot\nabla\phi_1)\mathbf{v}_V) \\
&= 0,
\end{aligned}
\tag{1}
$$

where $\mathbf{v}_V$ (background velocity), ρ_V (mass density) and c_V (speed of sound) will be considered as the acoustic properties of virtual space. There are two reasons behind the choice of Eq. (1). First, although it is not form invariant under general spacetime transformations either, there is a well-known relativistic model that is analogue to this equation [33,34]. Second, Eq. (1) allows us to consider the propagation of waves in a moving fluid, which provides the missing degrees of freedom.

If we directly applied a coordinate transformation mixing space and time to Eq. (1), there would appear new terms that could not be ascribed to any property of the medium. The ATA method starts by momentarily interpreting this equation as a different one with better transformation properties. In particular, we use the fact that Eq. (1) can be written as the massless Klein–Gordon equation of a scalar field ϕ_1 propagating in a (3+1)-dimensional pseudo-Riemannian manifold (the