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Dynamical Green's function theory to study the optical phenomena related to metamaterials

Weihua Wang, Xueqin Huang, Lei Zhou *

Surface Physics Laboratory (State Key Laboratory) and Physics Department, Fudan University, Shanghai 200433, PR China

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Abstract

We combine a space–time Lorentz transformation with a dyadic Green's function technique to establish a general and rigorous dynamical theory, which can be employed to study the optical phenomena occurring in a moving environment. As the applications of this method, we studied the Doppler effects of a source moving near a metamaterial slab and the super imaging effect achieved by a moving metamaterial lens. Many interesting anomalous phenomena were discovered, induced typically by the interplays between the source and the surface waves of the metamaterial slab.

Keywords: Metamaterials; Doppler effect; Super lensing; Green's function; Surface wave

1. Introduction

Metamaterials, artificial electromagnetic (EM) materials constituted by subwavelength local resonant structures, have attracted intensive attention recently [1–3]. Engineered by the microstructures which can provide electric and/or magnetic resonances, the entire medium can exhibit arbitrary values of effective permittivity ε and permeability μ. In particular, when both ε and μ are negative, the medium exhibits a negative refractive index [1–4], so that it is sometimes termed as a left-handed material [1–4]. Many unusual optical effects, which are typically unrealizable with conventional materials, were proposed or realized based on metamaterials. These include the negative refraction [1–5], super lensing effect [5–7], inverse Doppler and Cerenkov effects [8–14], extraordinary photonic band gap effects [15,16], and even the invisible cloaking [17,18], etc.

In reviewing these previous studies, we found that while there were quite a lot of efforts in studying the metamaterial-related optical effects generated by a *static* radiating source [5–7,15–18], the theoretical developments are still not satisfactory for the situations where the sources and medium are in relative motions. For instance, to study the Doppler effects, some previous theoretical studies employed a *Galileo* transformation to treat the movements of the source and receiver, and considered a plane wave source which is not so realistic [8–14]. On the other hand, it is also interesting to ask how those intriguing phenomena, such as the super lensing [5–7] and the invisible cloaking effects [17,18], evolve when the source is in relative motion with respect to the medium. Such problems are highly nontrivial since the metamaterials must be frequency dispersive due to the causality requirement [19], and therefore, their responses to a moving source could be rather intriguing and may generate many unexpected new phenomena.

* Corresponding author.
E-mail address: phzhou@fudan.edu.cn (L. Zhou).

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The motivation of this paper is to set up a general and rigorous theoretical framework to study the metamaterial-related optical effects in the cases when the source, receiver, and the medium are in relative motions. We will combine the *Lorentz* space–time transformation [20,21] with the time-dependent dyadic Green's function method [21,22] to achieve this goal. We have already employed a special form of the general approach to study the Doppler effects of a light source moving on the surface of a dispersive metamaterial slab [23]. Here, we present all the details of mathematic derivations of the general theory, and employ the theory to study some other intriguing optical effects related to moving source/medium.

The present paper is organized as follows. We first establish the theoretical framework in the next section, and present some benchmark results in Section 3. We then apply the theory to study the anomalous Doppler effects and the super lensing effect in Section 4. We finally conclude our paper in Section 5.

2. The dynamical theory for moving source/medium

We take a particular situation as an example to establish the theoretical framework, and the theory can be generalized to more complicated situations. As shown in Fig. 1, we consider the cases that the source and receiver are moving at different velocities, $\hat{v}_s = v_s(\cos\theta_s\hat{e}_x + \sin\theta_s\hat{e}_z)$, $\hat{v}_r = v_r(\cos\theta_r\hat{e}_x + \sin\theta_r\hat{e}_z)$, with respect to a metamaterial slab placed on the xy plane lying between $z = -0.5d$ and $z = -1.5d$ (We set $d = 10$ mm in all our calculations). We assume that the permittivity and permeability of the metamaterial slab are given by $\varepsilon_r(\omega)$, $\mu_r(\omega)$. Since both the source and receiver are moving, we define three frames—S' frame where the source is static, S frame where the slab is static and $\tilde{S}$ frame where the receiver is static. As usual, we assume that the origins of these three frames coincide at $t' = t = \tilde{t} = 0$. For simplicity, our source is assumed as a line current source taking the following

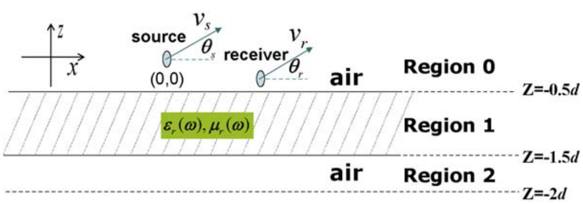

Fig. 1. Schematic shown of the studied geometry.

form

$$\vec{J}'(\vec{r}', t') = \hat{e}_y I_0 \delta(x')\delta(z') e^{-i\omega_0 t'} \theta(t'). \tag{1}$$

The source is located at the origin in the S' frame (See Fig. 1) and start to emit EM signals at $t' = 0$ [Noting that there is a $\theta(t')$ function in Eq. (1)]. Apparently, there is no charge accumulation in the line source, i.e., $\rho'(\vec{r}', t') = 0$. Our goal is to find the field distribution in the $\tilde{S}$ frame so that we can get the signal measured by the receiver. We will achieve this goal following three steps:

(1) We first calculate the EM fields radiated from the source in the S' frame, assuming at the moment that there is no metamaterial slab;
(2) we next employ a Lorentz transformation to transform such EM fields from the S' frame to S frame, then "put" the metamaterial slab into the frame and calculate the scatterings of these EM waves by the metamaterial slab;
(3) we finally apply another Lorentz transformation to obtain the entire field distributions in the $\tilde{S}$ frame.

2.1. Incident EM waves observed in the S' frame

In this subsection, let us "remove" the metamaterial slab for a moment. In the S' frame where the source is static, EM waves satisfy

$$\left| \begin{array}{l} \left[\nabla\times\nabla\times + \mu_0\varepsilon_0 \dfrac{\partial^2}{\partial t'^2}\right]\vec{E}'(\vec{r}', t') = -\mu\dot{\vec{J}}'(\vec{r}', t') \\ \left[\nabla\times\nabla\times + \mu_0\varepsilon_0 \dfrac{\partial^2}{\partial t'^2}\right]\vec{H}'(\vec{r}', t') = \nabla\times\vec{J}'(\vec{r}', t') \end{array} \right. \tag{2}$$

Define a dyadic Green's function by

$$\left(\nabla\times\nabla\times + \mu_0\varepsilon_0\frac{\partial^2}{\partial t'^2}\right)\overleftrightarrow{G}(\vec{r}', \vec{r}'_1; t', t'_1) = \delta(\vec{r}' - \vec{r}'_1)\delta(t' - t'_1)\overleftrightarrow{I}, \tag{3}$$

we found that

$$\begin{cases} \vec{E}'(\vec{r}', t') = -\mu_0 \displaystyle\int \overleftrightarrow{G}(\vec{r}', \vec{r}'_1; t', t'_1)\cdot\dot{\vec{J}}'(\vec{r}', t')d\vec{r}'_1 dt'_1 \\ \vec{H}'(\vec{r}', t') = \displaystyle\int \overleftrightarrow{G}(\vec{r}', \vec{r}'_1; t', t'_1)\cdot\left[\nabla\times\vec{J}'(\vec{r}', t')\right]d\vec{r}'_1 dt'_1 \end{cases}, \tag{4}$$