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Nonlocal magnetic configuration controlling realized in a triple-quantum-dot Josephson junction

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HIGHLIGHTS

- We focus on a triple-dot Josephson junction with central dot coupled to superconductors.
- Supercurrent exhibits phase translations due to the presence of spin and electron correlations.
- When side dots are half-occupied, nonlocal spin correlation coincides with supercurrent phase.
- We consider this system to be a candidate for controlling the nonlocal magnetic configuration.

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ABSTRACT

We investigate the Josephson effect in a superconductor/triple-quantum-dot/superconductor junction in which the central dot is coupled to the superconductors. It is found that the supercurrent exhibits rich 0–π phase translations due to the interplay between interdot spin and electron correlations. Moreover, when the side dots are half-occupied, the nonlocal spin correlation between them, i.e., ferromagnetic or antiferromagnetic, coincides well with the supercurrent phase. We thus consider such a system to be a promising candidate for controlling the nonlocal magnetic configuration based on the Josephson effect.

1. Introduction

Quantum dots (QDs) are highly tunable mesoscopic structures. Since their charge and spin degrees of freedom can be controlled, many significant physics phenomena are revealed, such as Coulomb blockade [1], Coulomb staircase [2], and the Kondo effect [3–6]. In comparison with a single-QD structure, the coupled-QD structures provide much more Feynman paths for electron transmission and possess more tunable parameters to manipulate electronic transport behaviors [4–10]. On the other hand, in the coupled-QD systems, the interdot spin–spin correlations play an important role in adjusting the competition between the Kondo effect and the interdot antiferromagnetic coupling. Thus, the coupled QDs provide a highly tunable platform for probing strong electronic correlation effects in artificial mesoscopic structures. More recently, many triple-QD (TQD) structures have been proposed for studying the competition between the Kondo effect and the interdot spin–spin correlations [11–15]. When the central QD connects with the external leads, Chiappe et al. reported that the side-QD spins can be coupled in the ferromagnetic or antiferromagnetic manners, determined by the occupation in the central QD [15].

It is also well known that when one QD is coupled to the superconducting leads, electron correlations can also induce various interesting phenomena, due to the interplay between the Josephson effect and electron correlations [16–25]. A paradigmatic example is the prediction about the π-junction behavior [16–19]. In such systems, the ratio of the Kondo temperature T_K to the superconducting gap Δ is a key parameter. In the strong-coupling limit where $T_K \gg \Delta$, the Kondo effect survives even in the presence of superconductivity, because a Cooper pair is broken in order to screen the localized spin in the QD. Alternatively, in the weak-coupling limit with $T_K \ll \Delta$, the Kondo effect is negligible due to the robustness of Cooper pair. And then, the Cooper pair feels the localized magnetic moment in the QD. Under this situation, when the Coulomb interaction is strong, the so-called 0–π transition takes place. Namely, the relation between supercurrent I_J and Josephson phase difference ϕ changes from $I_J = I_c \sin\phi$ to $I_J = I_c \sin(\phi + \pi) = -I_c \sin\phi$. Next, for coupled-QD geometries, the

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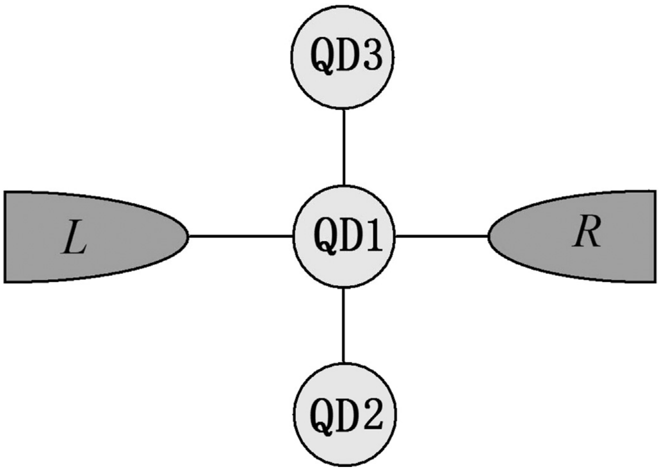

Fig. 1. Schematic of a S/TQD/S structure. QD-1 is connected to two s-wave superconductors, and QD-2 and QD-3 are side-coupled to QD-1 simultaneously.

situation will be more complicated due to the interplays among the Kondo effect, the interdot exchange interaction, and pairing correlations. As a result, there are three different energy scales, i.e., Kondo temperature T_K, interdot antiferromagnetic exchange interaction J, superconducting gap Δ. One can thus be sure that compared with the single-QD structure, richer magnetic-related behaviors can be realized in coupled-QD Josephson junctions.

In the present work, we would like to take the TQD Josephson junction as an example to investigate the interplay between Josephson effect and spin correlations. The geometry of the TQD structure is illustrated in Fig. 1, in which the side QDs, labeled as QD-2 and QD-3, are coupled to the central QD (i.e., QD-1) via a direct hopping term t_c. Meanwhile, QD-1 is connected with two s-wave superconductors. Via calculation, we found the 0–π phase-transition behaviors which are caused by the competition among the Kondo effect, the interdot exchange interaction, and pairing correlations. Moreover, when the side QDs are half-occupied, the nonlocal spin correlation between them can be ferromagnetic or antiferromagnetic, which depends on the central-QD level and Josephson phase difference. What is notable is that the spin correlation mode is consistent with the supercurrent phase. Therefore, this system provides an alternative scheme to control the nonlocal magnetic configuration by Josephson effect.

2. Theory

The Hamiltonian of the S/TQD/S structure is written as $H = \sum_\alpha H_\alpha + H_D + H_T$ where

$$H_\alpha = \sum_{k\sigma} \varepsilon_{\alpha k} a^\dagger_{\alpha k\sigma} a_{\alpha k\sigma} + \sum_k (\Delta e^{i\varphi_\alpha} a_{\alpha k\downarrow} a_{\alpha -k\uparrow} + \Delta e^{-i\varphi_\alpha} a^\dagger_{\alpha -k\uparrow} a^\dagger_{\alpha k\downarrow}),$$
$$H_D = \sum_{\sigma,j=1}^{3} \varepsilon_j d^\dagger_{j\sigma} d_{j\sigma} + \sum_\sigma (t_c d^\dagger_{2\sigma} d_{1\sigma} + t_c d^\dagger_{3\sigma} d_{1\sigma} + h.\,c.\,) + \sum_{j=1}^{3} U_j n_{j\uparrow} n_{j\downarrow},$$
$$H_T = \sum_{\alpha k\sigma} (V_{\alpha k} a^\dagger_{\alpha k\sigma} d_{1\sigma} + h.\,c.\,). \tag{1}$$

H_α ($\alpha = L, R$) is the standard BCS mean-field Hamiltonian for superconductor-α with phase φ_α and energy gap Δ. H_D models the TQDs, and H_T denotes the tunneling between superconductor-$L(R)$ and QD-1. $a^\dagger_{\alpha k\sigma}$ and $d^\dagger_{j\sigma}$ ($a_{\alpha k\sigma}$ and $d_{j\sigma}$) are operators to create (annihilate) an electron with momentum k and spin orientation σ in superconductor-α and in QD-j, respectively. $\varepsilon_{\alpha k}$ and ε_j denote the corresponding energy levels. U_j indicates the strength of intradot Coulomb repulsion, and t_c is the interdot coupling coefficient. $V_{\alpha k}$ denotes the coupling between superconductor-α and QD-1.

In such a structure, the Josephson current at zero temperature can be evaluated by deriving the ground-state (GS) energy E_{GS} with respect to the superconducting phase difference, i.e.,

$$I_J = \frac{2e}{\hbar}\frac{\partial E_{GS}(\varphi)}{\partial \varphi}, \tag{2}$$

where $\varphi = \varphi_L - \varphi_R$. However, note that the determination of the ground state is a formidable task, which requires some approximation schemes. A great simplification can be made by integrating out the electronic degrees of freedom of the superconductors. This procedure leads to an effective low-energy theory in which each superconductor is replaced by a single site with an effective pairing potential $\tilde{\Delta}$, and the hopping term V_α is replaced by an effective parameter $\tilde{V}_\alpha$. Accordingly, the new expressions of H_α and H_T are given by

$$H_\alpha = \sum_\sigma \varepsilon_\alpha a^\dagger_{\alpha\sigma} a_{\alpha\sigma} + \tilde{\Delta} e^{i\varphi_\alpha} a_{\alpha\downarrow} a_{\alpha\uparrow} + \tilde{\Delta} e^{-i\varphi_\alpha} a^\dagger_{\alpha\uparrow} a^\dagger_{\alpha\downarrow},$$
$$H_T = \sum_\sigma (\tilde{V}_L a^\dagger_{L\sigma} d_{1\sigma} + \tilde{V}_R a^\dagger_{R\sigma} d_{1\sigma} + h.\,c.\,). \tag{3}$$

Such an approach, usually referred to as the zero-bandwidth model (ZBWM), has been extensively used to investigate the supercurrent properties in the S/QD/S structures [16–18]. It has been shown that the ZBWM can give qualitatively correct results and can grasp the leading GS properties in this kind of geometries in the approximate range $\Gamma \le \Delta$, where Γ is the standard tunneling rate between the QD and superconductors. We would like to mention that within the ZBWM, the Hilbert space of the new system is restricted to 4^5 states and the z component of the total spin $\mathbf{S}$ is a good quantum number. Thus, the eigenstates can be characterized in terms of S_z and the eigenenergies can be obtained by the block diagonalization of the Hamiltonian matrix.

With respect to the supercurrent, it is well known that its phase can be distinguished according to the relation between E_{GS} and φ. If the GS energy as a function of φ has a global minimum at the point of $\varphi=0$ ($\varphi=\pi$), the current will be located as its 0 (π) phase. For the 0′ (π') phase, it describes the case where one local minimum emerges at the point of $\varphi=\pi$ ($\varphi=0$) in the E_{GS} spectrum, in addition to the global minimum at $\varphi=0$ ($\varphi=\pi$) [26,27].

3. Numerical results and discussions

With the formulas developed above, we next perform numerical calculations to investigate the characteristics of the Josephson effect in the S/TQD/S structure, by adopting the ZBWM. In principle, the effective parameters $\tilde{\Delta}$ and $\tilde{V}_\alpha$ in this approach have to be determined from the bare parameters Δ and V_α by means of a self-consistency condition and using a renormalization group analysis, as discussed by Affleck et al. [19]. We shall adopt here the simplifying assumption that $\tilde{\Delta} = \tilde{V}_\alpha = 1$ without an attempt to explore the detailed parameter relations, since we are mainly interested in the qualitative trends rather than the detailed quantitative results. The above assumption indicates that all the energy quantities in this work are scaled by $\tilde{\Delta}$. Besides, in order to get the main physical insight, we set the Fermi energy to be the energy zero point. As for the other parameters, we take $\phi_L = -\phi_R = \phi/2$, $\varepsilon_j = \varepsilon_0$, and $U_j = U$ unless otherwise specified. In the context, the uniform interdot coupling t_c is taken to be t_c=4.0 for calculation.

In Fig. 2(a), we plot the (ε_0, U) phase-transition diagram. In this figure, one can find that in the limit of $U \to 0$, no 0–π transition phenomenon takes place. The increase of Coulomb strength leads to the occurrence of the 0–π transition because of the presence of electron correlation. Moreover, there appear three transition regions and they are manifested as electron-hole symmetry about