Physica E 81 (2016) 77–82

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Optimal control-based states transfer for non-Markovian quantum system

Ji Ying-Hua [a,b,*], Hu Ju-ju [a,b], Huang Jian-Hua [a,b], Ke Qiang [a,b]

[a] College of Physics and Communication Electronics, Jiangxi Normal University, Nanchang, Jiangxi 330022, China
[b] Key Laboratory of Photoelectronics and Telecommunication of Jiangxi Province, Nanchang, Jiangxi 330022, China

ARTICLE INFO

Article history:
Received 26 October 2015
Accepted 29 February 2016
Available online 2 March 2016

Keywords:
Quantum optimal control
Non-Markovian open quantum systems
State transfer
Decoherence

ABSTRACT

Utilizing the method of optimal control, we investigate the tactics of state transfer in the non-Markovian quantum system with phase relaxation and energy dissipative relaxation. The influence of Ohmic reservoir with Lorentz–Drude regularization is numerically studied. Owing to the decoherence and memory effects of non-Markovian channel, the purity of quantum state attenuates damply in the free evolution. The numerical simulations indicate that arbitrary state transfer for non-Markovian system can be realized under the optimal control function by a proper external control field with a success rate of more than 98 percent. When the right control field and function is implemented, not only the decoherence is compensated completely but also the purity of quantum states are maintained in the process of state transfer.

1. Introduction

An important task in system control is designing efficient control laws for given systems by proper control theory so as to manipulate the initial state into the expected target state. Quantum information technologies are composed of quantum computation, quantum communication and quantum control [1,2] Combining the principles of quantum mechanics and approaches of classical engineering control, quantum control implement controlling on the states of quantum system by means of electrical field, magnetic field or electromagnetic field. The technologies of quantum control have been the effective way for maintaining the coherence and entanglement of the quantum system [3,4]. For the closed quantum system, people adopt the strategies of open-loop control, optimal control, learning control and feedback control to realize quantum control, and the research results achieved have successfully been applied in quantum information, quantum chemistry, laser cooling and new nanotechnology, etc [5].

Any realistic physical system will suffer from unwanted interactions with the outside environments, causing decoherence and destroying entanglement [6]. In order to investigate the quantum decoherence and the entanglement dynamics of open quantum systems, we must take into account the characteristics of the environments coupled to the system. The interaction between quantum system and the environment can be divided into Markovian and non-Markovian processes. For the Markovian process, information transferred from the quantum system to the environment will not return to the system from the environment. Considering memory effects in the non-Markovian process, the information previously transferred from a quantum system to the environment will provide feedback to system dynamics after a period of time, thereby influencing the next stage of system evolution. Thus, in the non-Markovian process, the dynamic behavior of the system will be more complex [7–9]. Usually, in the standard theory of the open quantum systems, environments are treated as the Markovian environments without memory. However, in the strong coupling regime, non-Markovian effects induced by the memory and feedback interactions of environments should be considered carefully. Recently, the strong coupling system for quantum information processing has been realized with the development of science and technology [10,11]. Thus, investigating the non-Markovian dynamics and related quantum coherent control of open quantum systems has important theoretical and practical significance.

So far, some research results have been achieved although the complete theories are not yet formed. The theory of optimal control is a well-developed field and finds numerous applications to the optimization of nonlinear and highly complex dynamic systems [12]. In the quantum chemistry context, optimal control was originally proposed by Rabitz and coworkers as a control scheme of reaction channels and was extensively used in various control experiments [13]. Optimal control theory provides a systematic and flexible formalism that can be used in quantum computation to generate reliable and high precision quantum

* Corresponding author at: College of Physics and Communication Electronics, Jiangxi Normal University, Nanchang, Jiangxi 330022, China.

http://dx.doi.org/10.1016/j.physe.2016.02.050
1386-9477/

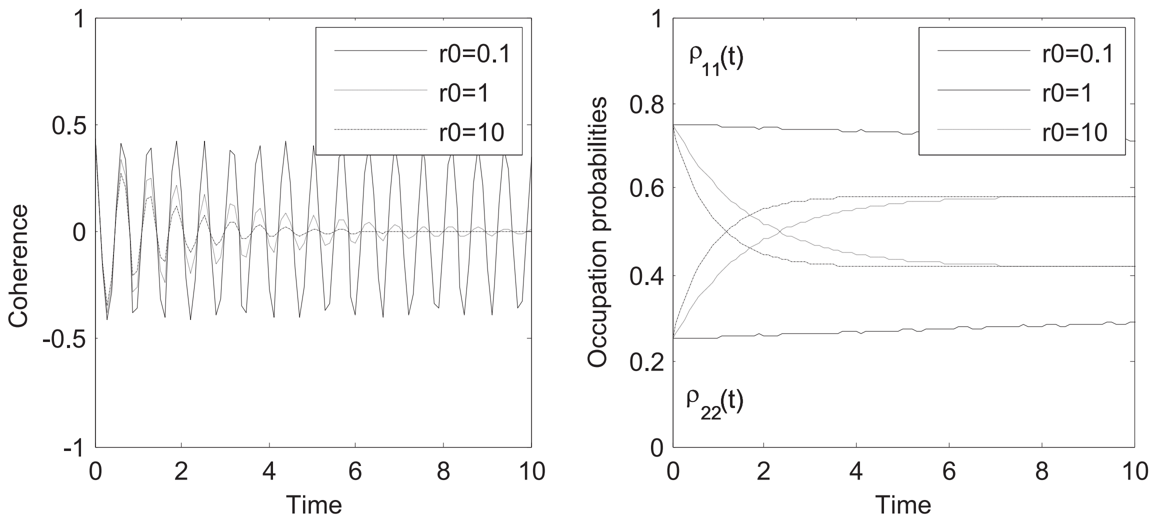

Fig. 1. Free evolutions of system without external control fields with $\alpha = 0.1$, $k_B T = 3\omega_0$.

dynamics [14]. A very recent application deals with the optimization of a Not-gate for superconducting phase qubits [15]. The optimal control plays significant roles in quantum open-loop control. Up to now, however, the research on decoherence suppression of open (non-Markovian) quantum system with the method of optimal control is seldom. Utilizing the thought of optimal control, we derive the control law of optimal control by taking the system state reaching the expectation value of the target state as the performance indicator. Then, we perform optimal control simulation experiments. The example of a two-level open non-Markovian quantum system validates its application in state transfer for non-Markovian system. The influence of the optimal control method on the system coherence and purity of quantum state is also investigated.

2. Model

The coherent control on quantum system is realized usually through interacting the externally applied optical field or electromagnetic field with the dynamical variables of the system, which is equivalent to introduce some Hamiltonian into the original Hamiltonian to change the energy of the system. In this paper, we only consider a singlet qubit subject to non-Markovian decoherence and controlled via a control Hamiltonian $H_C(t)$. Then, the Hamiltonian without dissipation can be written as ($k_B = \hbar = 1$):

$$H(t) = H_S + H_C(t), \tag{1}$$

with

$$\hat{H}_S = -\frac{1}{2}\varepsilon\,\hat{\sigma}_z - \frac{1}{2}\Delta\,\hat{\sigma}_x, \tag{2}$$

$$H_C(t) = \sum_k u_k(t)\,\sigma_k, \tag{3}$$

where σ_k with $k = x,\ y,\ z$ are the Pauli matrices; ε and Δ are production related parameters, which are constant in this paper; $u_k(t)$ is the modulation by the time-dependent external control field. In fact, the free Hamiltonian is H_S, which is widely used to describe the solid state qubits such as superconducting qubits, semiconductor dot etc [16,17]. The control Hamiltonian can be described by $\{\sigma_x,\ \sigma_y,\ \sigma_z\}$ according to Cartan decomposition of the Lie algebra $su(2)$, which was discussed in detail. [18]

According to the quantum dissipation theory, in the non-Markovian process, the reduced density $\rho(t)$ of the coupling quantum system satisfies the following motion equation

$$\frac{d}{dt}\rho(t) = \frac{1}{i\hbar}[H(t),\ \rho(t)] + L\rho(t), \tag{4}$$

with

$$\begin{aligned} L\rho(t) = {} & \frac{\Delta(t)+\gamma(t)}{2}\left(\left[\sigma_-\rho,\quad \sigma_-^+\right] + \left[\sigma_-,\quad \rho\sigma_-^+\right]\right) \\ & + \frac{\Delta(t)-\gamma(t)}{2}\left(\left[\sigma_+\rho,\quad \sigma_+^+\right] + \left[\sigma_+,\quad \rho\sigma_+^+\right]\right) \\ & + \frac{\gamma(t)}{2}\left(\left[\sigma_z\rho,\quad \sigma_z^+\right] + \left[\sigma_z,\quad \rho\sigma_z^+\right]\right), \end{aligned} \tag{5}$$

where

$$\Delta(t) = \int_0^t d\tau k(\tau)\cos(\omega_0\tau), \tag{6}$$

$$\gamma(t) = \int_0^t d\tau\mu(\tau)\sin(\omega_0\tau), \tag{7}$$

$$k(\tau) = 2\int_0^\infty d\omega J(\omega)\coth\left[\hbar\omega/2k_B T\right]\cos(\omega\tau), \tag{8}$$

$$\mu(\tau) = 2\int_0^\infty d\omega J(\omega)\sin(\omega\tau). \tag{9}$$

$\omega_0 = \sqrt{\varepsilon^2 + \Delta^2}$.$\gamma(t)$ is the dissipation coefficient, $\Delta(t)$ diffusion coefficient; $J(\omega)$ spectral density of environment.

Eq. (4) is the spread of Lindblad master equation in non-Markonvian system. Distinguishing from that in Markovian system, the dissipative coefficient of the system varies with time and can be positive or negative. Negative dissipative coefficient indicates that the information of open system is recovered or compensated, exemplifying the memory effect of non-Markovian system.

In this paper we discuss the question only in the Ohmic environment. Therefore, we choose the Ohmic spectral density with a Lorentz–Drude cut-off function

$$J(\omega) = \frac{2\gamma_0}{\pi}\omega\frac{\omega_c^2}{\omega_c^2 + \omega^2}, \tag{10}$$

where γ_0 is the frequency-independent damping constant, ω is the frequency of the bath, and ω_c is the high-frequency cutoff. The parameters $\gamma(t)$ and $\Delta(t)$ contain the non-Markovian features of