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Magnetization of interacting electrons in anisotropic quantum dots with Rashba spin–orbit interaction

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ABSTRACT

Magnetization of anisotropic quantum dots in the presence of the Rashba spin–orbit interaction has been studied for three and four interacting electrons in the dot for non-zero values of the applied magnetic field. We observe unique behaviors of magnetization that are direct reflections of the anisotropy and the spin–orbit interaction parameters independently or concurrently. In particular, there are saw-tooth structures in the magnetic field dependence of the magnetization, as caused by the electron–electron interaction, that are strongly modified in the presence of large anisotropy and high strength of the spin–orbit interactions. We also report the temperature dependence of magnetization that indicates the temperature beyond which these structures due to the interactions disappear. Additionally, we found the emergence of a weak sawtooth structure in magnetization for three electrons in the high anisotropy and large spin–orbit interaction limit that was explained as a result of merging of two low-energy curves when the level spacings evolve with increasing values of the anisotropy and the spin–orbit interaction strength.

Magnetization of quantum confined planar electron systems, e.g. quantum dots (QDs), the so-called artificial atoms [1–3], and quantum rings [4] is an important probe that manifests entirely on the properties of the energy spectra. This is a thermodynamical quantity that for the QDs has received some experimental attention [5–7], particularly after the theoretical prediction that the electron–electron interaction is directly reflected in this quantity [8]. In addition to the large number of theoretical studies reported in the literature on the electronic properties of isotropic quantum dots, there has been lately some studies on the *anisotropic* quantum dots, both theoretically [10,11] and experimentally [12]. Theoretical studies of the magnetization of elliptical QDs have also been reported [13]. Effects of the Rashba spin–orbit interaction (SOI) [14] on the electronic properties of isotropic [15] and anisotropic quantum dots [16] have been investigated earlier. An external electric field can induce the Rashba spin–orbit interaction [17] which couples different spin states and introduces level repulsions in the energy spectrum [15,16,18]. This coupling is an important ingredient for the burgeoning field of semiconductor spintronics, in particular, for quantum computers with spin degrees of freedom as quantum bits [19,20]. Three-electron quantum dots are particularly relevant in this context [21,22]. Electronic properties of parabolic quantum dots, including magnetization, was reported recently in the case of the ultrastrong Rashba SO coupling limit [23]. Here we report on the magnetic field dependence of the magnetization of an anisotropic QD containing several interacting electrons, particularly three and four, in the presence of the Rashba SOI. Our present work clearly demonstrates how magnetization of the QDs uniquely reflects the influence of anisotropy and the Rashba SOI, both concurrently as well as individually as the strengths of the SOI and the anisotropy are varied independently. The temperature dependence of magnetization is also studied here, where we noticed the gradual disappearance of the interaction induced structures in magnetization with increasing temperature. Another important feature that we found in our present study is the emergence of a weak sawtooth structure in three-electron magnetization result in the high anisotropy and large spin–orbit interaction limit that we explain as a result of merging of two low-energy curves when the level spacings evolve with increasing parameters. With the help of the theoretical insights presented here, experimental studies of magnetization will therefore provide valuable information on the inter-electron effects, the Rashba spin–orbit coupling and the degree of anisotropy of the quantum dots.

At zero temperature the magnetization $\mathcal{M}$ of the QD is defined as $\mathcal{M} = -\frac{\partial E_g}{\partial B}$ where E_g is the ground state energy of the system [8,9]. We have studied the magnetic field dependence of $\mathcal{M}$ by evaluating the expectation value of the magnetization operator $\hat{m} = -\frac{\partial \mathcal{H}}{\partial B}$, where $\mathcal{H}$ is the system Hamiltonian. Since the Coulomb

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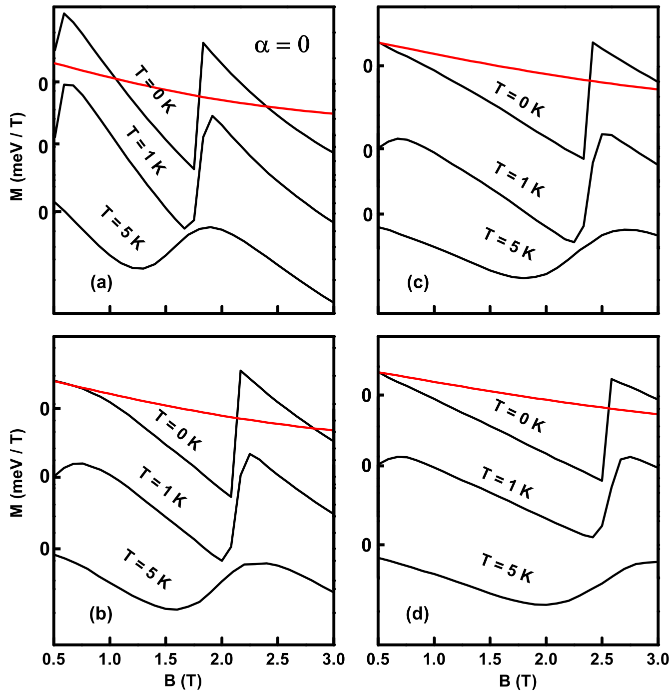

Fig. 1. Temperature dependence of magnetization of a three-electron anisotropic dot without the Rashba SOI ($\alpha = 0$). The results are for $\omega_x = 4$ meV and (a) $\omega_y = 4.1$ meV, (b) $\omega_y = 6$ meV, (c) $\omega_y = 8$ meV, and (d) $\omega_y = 10$ meV. The zero-temperature magnetization curve for the non-interacting system is also shown in red. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

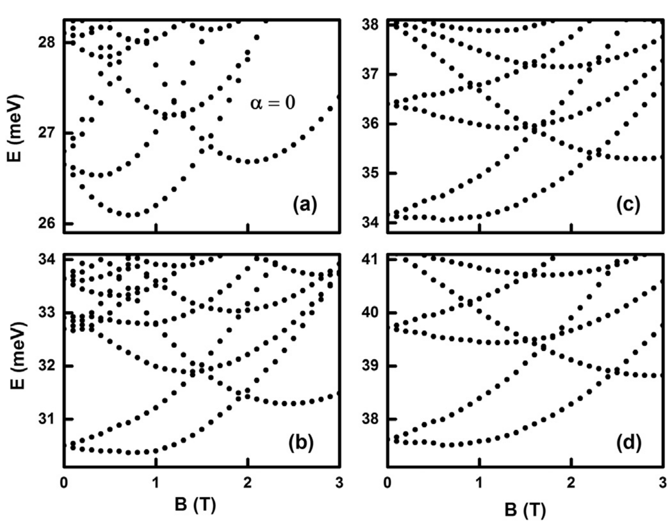

Fig. 2. Energy levels of a three-electron anisotropic dot without the Rashba SOI ($\alpha = 0$). The results are for $\omega_x = 4$ meV and (a) $\omega_y = 4.1$ meV, (b) $\omega_y = 6$ meV, (c) $\omega_y = 8$ meV, and (d) $\omega_y = 10$ meV.

interaction is independent of B, $\hat{m}$ would be just a one-body operator, i.e., we can ignore the interaction part from the Hamiltonian. The Hamiltonian of a single-electron system subjected to an external magnetic field with the vector potential $\mathbf{A} = \frac{1}{2}B(-y, x)$, the confinement potential, and the Rashba SOI is

$$\mathcal{H} = \frac{1}{2m_e}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 + \frac{1}{2}m_e\left(\omega_x^2 x^2 + \omega_y^2 y^2\right) + \frac{\alpha}{\hbar}\left[\boldsymbol{\sigma} \times \left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)\right]_z + \frac{1}{2}g\mu_B B\sigma_z.$$

The first term of the Hamiltonian is the kinetic energy, which can

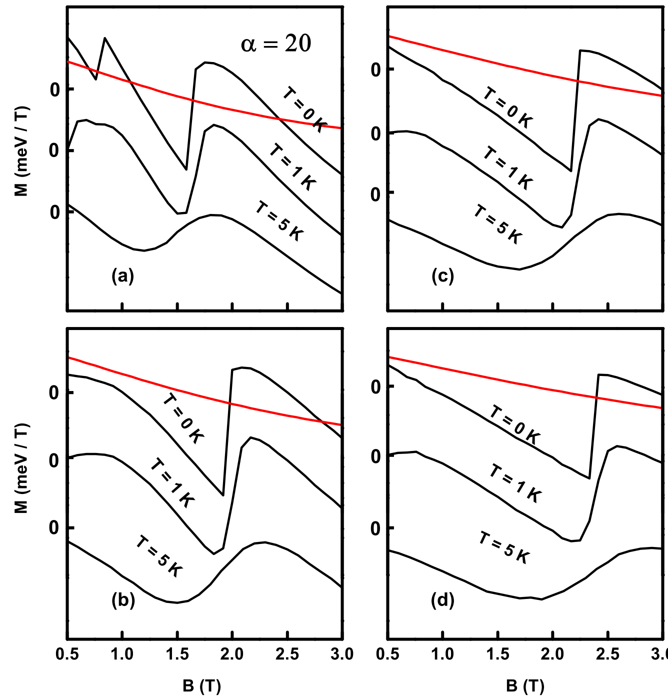

Fig. 3. Same as in Fig. 1, but for $\alpha = 20$ meV nm. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

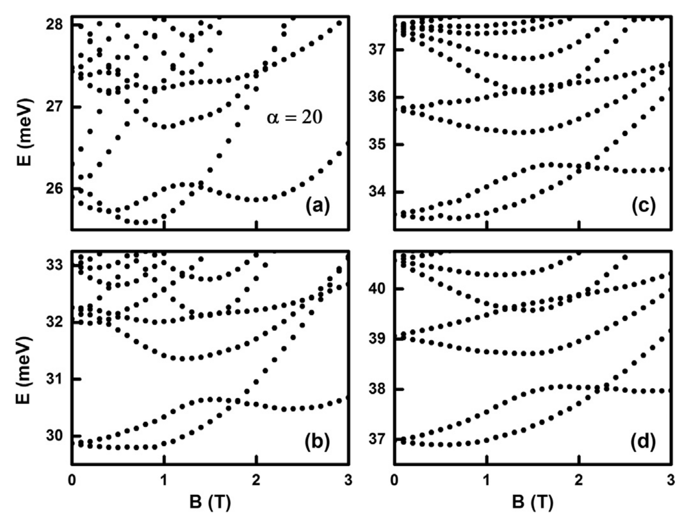

Fig. 4. Same as in Fig. 2, but for $\alpha = 20$ meV nm.

be written as

$$K = \frac{1}{2m_e}\left(p_x^2 + p_y^2 + \frac{eB}{c}(yp_x - xp_y) + \frac{e^2B^2}{4c^2}(y^2 + x^2)\right).$$

The SOI part (third term) is

$$H_{SO} = \frac{\alpha}{\hbar}\left[\sigma_x\left(p_y - \frac{eB}{2c}x\right) - \sigma_y\left(p_x + \frac{eB}{2c}y\right)\right],$$

while the second and the last term correspond to the confinement potential and the Zeeman term, respectively. We then need to evaluate the expectation value of the magnetization operator

$$\hat{m} = -\frac{\partial H}{\partial B} = -\frac{1}{2m_e}\frac{e}{c}\left((yp_x - xp_y) + \frac{eB}{2c}(y^2 + x^2)\right) + \frac{e\alpha}{2c\hbar}(\sigma_x x + \sigma_y y) - \frac{1}{2}g\mu_B\sigma_z,$$