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Reprint of : Current drag in two leg quantum ladders

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H I G H L I G H T S

- Study of coupled superconducting wires or bosonic tubes.
- Determination of the induced in one wire by the other.
- Connection of this problem to the commensurate–incommensurate transition.

A R T I C L E I N F O

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A B S T R A C T

A two-leg ladder of either interacting bosons or tightly bound cooper pairs is investigated when a supercurrent is forced in one of the legs of the ladder. The two legs of the ladder are connected by a tunneling term. Using a bosonization representation of such an interacting ladder we show that up to a certain critical current the current in the first wire induces an identical supercurrent in the second wire. When this threshold is exceeded vortices are formed in the system and the current in the second wire reduces even if the driving current increases. Potential applications to condensed matter or cold atomic systems are discussed.

1. Introduction

Transport in mesoscopic systems has been proven to be a cornucopia of physical phenomena. A milestone in that field was the Laudauer–Buttiker (LB) formulation [1,2] that allowed to show that the transport was in fact connected, for non-interacting systems to the actual transmission and reflection of waves across the system and paved the way to understanding phenomena such as the quantization of conductance. This formalism has proven to be a key tool in our understanding of transport in mesoscopic structures [3].

The situation becomes unfortunately more intricate when interactions are present. In that case one cannot consider the transmission of each wave independently and thus the LB formalism is not directly applicable and with it gone so goes most of our understanding of transport in such mesoscopic structures. Several extensions of the formalism for interacting systems have been studied [4] but the general problem is hard to solve. One situation in which we can make progress is provided by one-dimensional systems [5]. In such systems the interactions can be taken care of by a variety of methods. In particular one method, known as bosonization, allows us to reduce the problem to an essentially non-interacting problem of collective excitations, allowing us to use methods similar to the LB formalism to study e.g. the conductance of the problem [6–8]. Of course, even in one dimension, interactions can also lead to problems which still are not reduced easily to noninteracting particles. This is in particular the case when scattering is present inside the one-dimensional structure [5].

Another remarkable case for which interactions play a central role even if the system is totally pure is when more than one one-dimensional structure is present. Probably the most well-known studied case corresponding to such a situation is the case of Coulomb drag where two structures (one- or two-dimensional) are close together. A current is forced in the first one and due to the Coulomb coupling to the second one can potentially induce either a current or a voltage in the second one. This problem has been extensively studied both theoretically and experimentally (see e.g. [9–11] and ref. therein).

In this paper we study a system close to idea of the Coulomb drag problem but for superconducting chains. We study a system

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of two superconducting (fermions with attractive interactions) or superfluid (bosonic) chains which are not coupled via interactions as in Coulomb drag but by a tunneling term of either cooper pairs (fermions) or single particles (bosons). In such a system one can force a supercurrent in the first wire and ask a similar question than for the Coulomb drag: What is the current induced in the second wire? Besides the interest from a theoretical point of view, the motivation for such a study is twofold. First: in cold atomic systems such ladders of bosonic tubes have been realized [12] and have proven excellent systems to tackle related phenomena such as the chiral edge states induced by an externally applied gauge field, equivalent to a magnetic field for charged particles. Second: in condensed matter, in addition to the more conventional superconducting wires available [13] it has been recently possible to write one-dimensional structures [14] on two-dimensional superconductors obtained at oxide interfaces [15] allowing us to potentially study the tunnelling between one-dimensional superconducting wires in the future.

The plan of the paper is as follows. In Section 2 we define the problem. Section 3 is a brief reminder of the bosonization technique that will be used in the solution. Section 4 discusses the solution and the main physical features. Section 5 is the conclusion.

2. Definition of the problem

We consider a two-leg ladder system as depicted in Fig. 1.

Each tube corresponds to either interacting one-dimensional bosons or fermions in the superconducting state. The Hamiltonian of the decoupled legs is thus

$$H^0 = H_1^0 + H_2^0 \tag{1}$$

where H_i^0 is the Hamiltonian of the interacting particles on leg i. For simplicity we ignore here the presence of an underlying lattice along the legs. This is realistic for realizations with cold atoms that can have bosonic tubes in a continuum. For the fermionic case of superconducting wires the (in)commensurability between the carrier density and the underlying microscopic lattice is so large that one can safely ignore the lattice. We will not specify the microscopic Hamiltonian H_0 for the moment since in both cases the low energy theory can be reduced to the same effective Hamiltonian (the so-called Tomonaga–Luttinger liquid (TLL) hamiltonian) that we will introduce in Section 3. The two legs are supposed to be coupled by a tunneling term. For bosons such coupling is of the form

$$H_D = -t_\perp \int dx (\psi_1^\dagger(x)\psi_2(x) + \text{h. c.}) \tag{2}$$

For the case of superconducting wires the situations are microscopically more complex since the tunneling term involves in principle single particles. It is thus of the form (2) (with an additional spin index for each spin species). However in the case where the wires are superconducting and the superconducting gap Δ_σ (see Section 3) is larger than the tunnelling $t_\perp$ it is easy to see that the single particle tunnelling is exponentially suppressed. The only surviving coupling is quite naturally the Josephson coupling between the two chains and is of the form

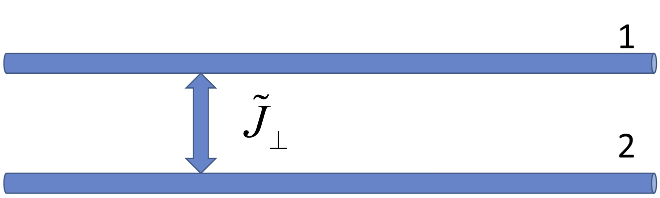

Fig. 1. A two-leg ladder made of two interacting one-dimensional systems coupled by a tunneling term of either cooper pairs (fermions) or single particles (bosons).

$$H_D = -J_\perp \int dx\, (\psi_{1\uparrow}^\dagger(x)\psi_{1\downarrow}^\dagger(x)\psi_{2\downarrow}(x)\psi_{2\uparrow}(x) + \text{h. c.}) \tag{3}$$

for which a pair is hopping from one chain to the next. The Josephson tunnelling $J_\perp$ is of the order of $t_\perp^2/\Delta_\sigma$.

Thus we will consider in the following a system described by:

$$H = H^0 + H_D \tag{4}$$

in which H_D is either (2) or (3) and study the interplay in the currents resulting from such Hamiltonian.

3. Bosonization reminder

In order to treat this problem we use the bosonized representation of interacting one-dimensional systems. The method is well documented [5] and we will only do a brief reminder here.

Let us treat first the case of bosons. The density of one-dimensional bosons can be expressed in term of a smooth field $\phi(x)$ as

$$\rho_j(x) = \rho_0 - \frac{1}{\pi}\nabla\phi_j(x) + \rho_0 \sum_p e^{i2p(\rho_0 x - \phi_j(x))} \tag{5}$$

where $j=1$, 2 is the chain index, p is an integer and ρ_0 is the average density which for simplicity we have assumed to be identical on the two legs. This expression of the density combined with an expression of the single particle creation operator as a function of another smooth field $\theta(x)$

$$\psi_j^\dagger(x) = \sqrt{\rho_j(x)}\, e^{-i\theta_j(x)} \tag{6}$$

provides a dictionary to go from the expressions of the Hamiltonian in terms of individual bosonic excitations to the collective ones that are density and phase (hence related to the current) excitations. Since one-dimensional interactions prevent the existence of individual excitations at the profit to collective ones, the new basis ϕ, θ is thus a much more convenient basis to use. Of course the ϕ and θ are not independent operators but are related by the canonical relation

$$[\phi(x), \frac{1}{\pi}\nabla\theta(x')] = i\delta(x - x') \tag{7}$$

The relation embeds at the operator level for one dimension the well known duality between phase fluctuations and density fluctuations that exists in any superfluid or superconductor. θ is thus related to $\Pi(x)$ the momentum conjugates to ϕ by

$$\Pi(x) = \frac{1}{\pi}\nabla\theta(x) \tag{8}$$

As a function of these new variables one can show that the Hamiltonian of a one-dimensional interacting bosonic system can be put under the form

$$H_j^0 = \frac{1}{2\pi}\int dx ((u_j K_j)(\nabla\theta_j(x))^2 + \left(\frac{u_j}{K_j}\right)(\nabla\phi_j(x))^2) \tag{9}$$

where all the interactions, bandstructure, etc. for the bosons have been absorbed in the two non-universal parameters (the so-called Luttinger parameters) u_j and K_j. u_j has the dimensions of a velocity and is the sound velocity of the excitations in the system. K_j is a dimensionless parameter that controls the decay of the correlation functions in the system. The form of the Hamiltonian (9) is the universal form of the so-called Tomonaga–Luttinger liquids that described the low energy physics of many one-dimensional systems. The value of the parameters depends not only on the interactions and kinetic energy but also on the class of problem considered (fermions, spins, bosons, range of interactions, etc.). For

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