Physica E 82 (2016) 85–91

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Reprint of : Waiting times of entangled electrons in normal–superconducting junctions

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ARTICLE INFO

Article history:
Received 30 June 2015
Received in revised form
24 September 2015
Accepted 29 October 2015
Available online 14 March 2016

Keywords:
Normal–superconducting junction
Andreev reflection
Waiting time distribution
Entanglement

ABSTRACT

We consider a normal–superconducting junction in order to investigate the effect of new physical ingredients on waiting times. First, we study the interplay between Andreev and specular scattering at the interface on the distribution of waiting times of electrons or holes separately. In that case the distribution is not altered dramatically compared to the case of a single quantum channel with a quantum point contact since the interface acts as an Andreev mirror for holes. We then consider a fully entangled state originating from splitting of Cooper pairs at the interface and demonstrate a significant enhancement of the probability to detect two consecutive electrons in a short time interval. Finally, we discuss the electronic waiting time distribution in the more realistic situation of partial entanglement.

1. Introduction

Markus Büttiker was certainly one of the most influential scientists in the field of mesoscopic physics. Among all his important contributions, time in quantum mechanics has a peculiar flavor since it occupied his mind at the right beginning and at the end of his carrier. Intrigued at first by the traversal time of an electron through a tunnel barrier [1,2], he came back to this topic after the emergence of "on-demand single electron sources" [3–13], which he greatly contributed to develop [14–23], via the concept of waiting time distribution (WTD) [24–26].

Charge transport at the nanoscale is known to be stochastic due to the quantum nature of particles [16]. Therefore, going beyond the knowledge of average quantities, such as the average electronic current, appears to be unavoidable and extremely fruitful at the same time. A deep physical insight can indeed be inferred from the fluctuations of the signal and extracted from various observables. Noise [16] and Full Counting Statistics (FCS) [27–29], namely the second moment of current fluctuations and the statistics of charges transferred during a long time interval, are among the most popular quantities and have been proved to be powerful tools. With the development of electron quantum optics [30] and the progress in single electron detection at high frequencies [4,31–33], it is now relevant and possible to consider electron dynamics and time resolved quantities at quantum mechanical time scales (typically nano-seconds and below). Therefore, new theoretical tools have been developed to describe the current fluctuations at such time scales, such as finite frequency noise [4,16,20,21,34–36] and FCS [37–42], Wigner functions [43], or the WTD [24–26,44–53]. The latter, describes the statistical distribution of time intervals between the detection of two electrons and therefore gives accurate information about correlations between subsequent electrons.

The WTD has been studied for particularly simple systems like single and multiple electronic quantum channels connected to two normal leads via a Quantum Point Contact (QPC) [25,49,52], a quantum capacitor [24,53], a double quantum dot [45,49], a train of Lorentzian pulses [26,47] or a quantum dot connected to a normal and a superconducting lead [48,54], among others. In this paper we revisit the physics of Normal–Superconducting (NS) junction through the point of view of waiting times in order to illustrate the effect of superconducting correlations and entanglement [55,56,59,60] on their distribution. Indeed, as we will discuss later, such a system may emit entangled electrons in the normal part, and leads to interesting features in the WTD.

The paper is organized as follows. In Section 2, we describe the model used for the NS junction and the formalism needed for computing the WTD. In Section 3, we discuss the effect of the transparency of the barrier (the energy dependence of the Andreev reflection) when the detection process is sensitive to only one electronic spin species and a certain range of energy. Section 4

DOI of original article: http://dx.doi.org/10.1016/j.physe.2015.10.033
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http://dx.doi.org/10.1016/j.physe.2016.02.017
1386-9477/© 2016 Published by Elsevier B.V.

is devoted to the effect of entanglement between spin up and spin down electrons emitted from the superconducting part, on the WTDs. We finally conclude and discuss some perspectives in Section 5. Moreover, for the sake of clarity, technical details are moved to the appendices. Appendix A demonstrates the formal analogy between our setup and a single quantum channel conductor for a specific detection process whereas important steps for the numerical and analytical calculations of the WTD in the entangled case are explained in Appendix B.

2. Model

One very important consequence of superconductivity is the existence of Andreev reflection. Such a phenomenon arises because the superconducting device cannot accommodate any single particle excitation with energy below the gap Δ. Therefore, if a single particle like an electron or a hole flows from the normal part to the superconducting part with an energy below this threshold it can only be scattered back at the interface. However, there are now two possibilities. An electron (a hole) can be either normally reflected (specular reflection), that is to say, reflected as an electron (a hole) or converted to a hole (an electron). This is the so-called Andreev reflection which originates from the fact that the incoming electron finds a partner to create a Cooper pair which can enter in the superconductor and leave a hole behind.

To be more specific, the system of interest is a polarized NS junction (with an s-wave superconductor), at zero temperature, as presented in Fig. 1. The superconductor chemical potential μ_S is set to be at a potential eV above the Fermi level E_F of the normal metal. In such a situation, there is an incident hole, coming from the metal, that can be either normally reflected or Andreev reflected as an electron. Another way of picturing the Andreev effect is to think about the inverse configuration where a Cooper pair in the superconductor (at energy μ_S and zero momentum for an s-wave superconductor) splits at the interface and gives birth to an entangled pair of electrons. From now on, we will take eV much smaller than the superconducting gap Δ in order to focus on this sub-gap phenomenon. This also has the benefit to make the Andreev time $t_A \equiv h/\Delta$ (the typical time needed for an Andreev event) much smaller than $\bar{\tau} \equiv h/(eV)$ (the typical time separation of two single particle wave packets emitted in the normal metal [25,61]). This allows us to assume that Andreev events are instantaneous and make use of scattering theory. In addition, this assumption allows one to linearize the dispersion relation around μ_S as $E(k) = \hbar v_F k$, with E and k measured from μ_S and its corresponding momentum (or around the Fermi level since $eV \ll E_F$ and μ_S).

At the interface, the scattering is in general not perfect and both normal and Andreev reflection will play a role. In order to describe this effect, we use the standard Blonder–Tinkham–Klapwijk (BTK) model [62] which has been widely used in the literature. The junction is modeled by a point-like barrier potential $U(x) = 2ZE_F\lambda_F\delta(x)$, where λ_F is the Fermi wavelength and Z is a parameter measuring the strength of the barrier. It is then possible to compute the scattering matrix of this setup exactly and obtain the normal and Andreev transmission/reflection coefficients [62,63]. We do not reproduce these results in the present paper but give the corresponding numerical values of the coefficients when necessary.

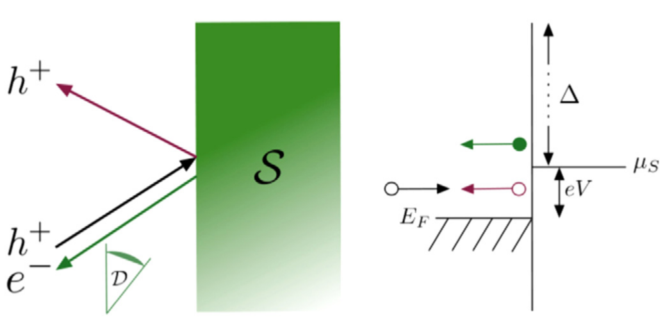

Fig. 1. Left: schematic picture of a Normal–Superconducting junction. A hole approaching the interface from the normal part is either normally reflected or Andreev reflected back as an electron. A single electron detector is positioned to detect electrons from Andreev events. Right: energy diagram of the setup. The superconducting chemical potential μ_S is set to an energy eV above the Fermi energy of the normal part and the gap Δ is much larger than the potential difference eV.

Fig. 1 illustrates the scattering processes that we are now going to describe mathematically. The incident holes of energies $\mu_S - E$ lying between E_F and μ_s, arriving from the left and propagating to the right will be either normally reflected as holes of the same energies with amplitude r_N or Andreev reflected as electrons of energies $\mu_S + E$ with amplitude r_A. The incoming scattering state is therefore a Slater determinant of holes of the form [57–59]

$$|\psi_{\rm in}\rangle = \prod_{E=0}^{eV} c_{k(E),\uparrow} c_{k(E),\downarrow} |0\rangle, \tag{1}$$

where $|0\rangle$ stands for the filled Fermi sea up to μ_S in the normal part. However, in the electron language, this state is just the Fermi sea $|F\rangle$ filled up to E_F instead of μ_S. In the following, we will rather use the electronic picture to simplify the notation but both pictures are equivalent [59]. Due to scattering at the interface, the outgoing state is therefore a superposition of reflected holes, entangled electrons and non-entangled electrons [56–59,62,63]

$$|\psi_{\rm out}\rangle = \prod_{E=0}^{eV} (r_N^*(E) + r_A(E) c^\dagger_{k(E),\uparrow} c^\dagger_{-k(-E),\downarrow}) \times (r_N(E) - r_A^*(E) c^\dagger_{k(E),\downarrow} c^\dagger_{-k(-E),\uparrow}) |F\rangle. \tag{2}$$

Indeed, it is pretty straightforward to see that the previous equation, for a given energy, gives birth to three kinds of term with different levels of complexity. The terms $|F\rangle$ and $c^\dagger_{k(E),\uparrow} c^\dagger_{-k(-E),\downarrow} c^\dagger_{k(E),\downarrow} c^\dagger_{-k(-E),\uparrow} |F\rangle$ correspond to non-entangled contributions whereas $(c^\dagger_{k(E),\uparrow} c^\dagger_{-k(-E),\downarrow} - c^\dagger_{k(E),\downarrow} c^\dagger_{-k(-E),\uparrow})|F\rangle$ describes fully entangled electrons originating from the splitting of a Cooper pair at the interface. When Andreev reflection is absent ($r_a = 0$), the Fermi Sea is unperturbed by the interface and nothing interesting happens. Counter-intuitively, perfect Andreev reflection does not lead to perfect entanglement. On the contrary, the state is a Slater determinant of non-entangled electrons and the NS junction acts as a conventional electron source [58]. It appears that the maximally entangled situation arises when Andreev and normal reflection probabilities are both equal to one half. Nevertheless, the WTD of a fully entangled state has never been studied to our knowledge and we will take the opportunity to study it in this paper before considering the general and more realistic state emitted at the interface.

In order to conclude this section, we recall a few definitions about WTDs. As mentioned in the introduction, the waiting time τ is defined as the time delay between the detection of two single particles. Due to scattering and the quantum nature of particles, this time is a random variable, in which distribution (the WTD) brings an elegant and instructive picture of the physics. For stationary systems, namely when there is no explicit time dependence, the WTD $\mathcal{W}(\tau)$ depends on τ only (and not on absolute time) and is closely related to the Idle Time Probability (ITP) $\Pi(\tau)$, the probability to detect no electron during a time interval τ

$$\mathcal{W}(\tau) = \langle\tau\rangle \frac{d^2\Pi(\tau)}{d\tau^2}, \tag{3}$$

where $\langle\tau\rangle = -[d\Pi(\tau)/d\tau]^{-1}_{\tau=0}$ is the mean waiting time [49]. To go further, we must now specify the detection procedure to compute the WTD. In what follows we will assume perfect single electron