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Reprint of : Cross-correlations of coherent multiple Andreev reflections

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H I G H L I G H T S

- We study current correlations in voltage-biased 3-terminal Josephson junctions.
- At low voltage, large and positive cross-correlations can be realized.
- The interplay between supercurrents and quasiparticle transport is nontrivial.

A R T I C L E I N F O

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A B S T R A C T

We use the Landauer–Büttiker scattering theory for electronic transport to calculate the current cross-correlations in a voltage-biased three-terminal junction with all superconducting leads. At low bias voltage, when charge transport is due to coherent multiple Andreev reflections, we find large cross-correlations compared with their normal-state value. Furthermore, depending on the parameters that characterize the properties of the scattering region between the leads, the cross-correlations can reverse their sign with respect to the case of non-interacting fermionic systems.

1. Introduction

Multiple Andreev reflections are the processes that explain how a dissipative charge transport can take place in a junction between superconducting leads at subgap voltages, $eV < 2\Delta$, where Δ is the superconducting gap, and low temperatures [1]. Indeed, due to the energy gap in the excitation spectrum of conventional superconductors, the direct transfer of a quasiparticle between the leads is not possible in that voltage range. However, a subgap electron incident on a superconducting lead can be Andreev reflected as a hole, while a Cooper pair is created in the lead [2]. By performing n successive Andreev reflections, it is possible to transfer $\sim n/2$ Cooper pairs – and one quasiparticle – between two superconductors at voltage bias $eV > 2\Delta/n$. This highly correlated process produces a rich subgap structure in the current–voltage characteristics, $I(V)$. Theoretically, this was predicted in incoherent [3] and coherent [4–6] ballistic junctions, as well as in diffusive junctions [7]. Experimentally, subgap structures in the $I(V)$-characteristics have been observed in a variety of systems, namely Josephson junctions based on exotic materials and nanoscale systems, such as semiconductor nanowires [8,9], carbon nanotubes [10–12], and graphene flakes [13]. Multiple Andreev reflections also result in a large shot noise, $S \sim q_* I$, both in the incoherent [14–16] and the coherent [17,18] regime. This effect can be ascribed to the divergence of the effective charge transferred in that process, $q_* = ne$ with $n \sim 2\Delta/(eV)$, as the voltage decreases. The enhancement of the shot noise at low voltage bias was observed in tunnel [19], metallic [20–22], and atomic point contact junctions [23]. In the context of topological superconductivity, multiple Andreev reflections were recently discussed as a parity-changing process for the Majorana bound state that is formed in a topological Josephson junction [24,25].

Further insight in the multiple Andreev reflection processes may be acquired through current cross-correlations in a multi-terminal geometry. As Markus Büttiker demonstrated in his seminal paper on shot noise, the cross-correlations in non-interacting fermionic systems are always negative due to the Pauli principle [26]. Later, several scenarios for a sign-reversal of the

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cross-correlations in the presence of interactions were proposed (see Ref. [27] for a review). For instance, the cross-correlations of the currents through two normal leads weakly contacted to a superconductor can be positive [28–36], due to a crossed Andreev reflection process in which an electron incident from one of the normal leads is Andreev-reflected to the other one [37–40]. The possibility to use such cross-correlations for a signature of entanglement, due to the singlet-state of the crossed Andreev pair, was also discussed [41]. Maximally positive cross-correlations – meaning that they are exactly opposite to the autocorrelation – in a topological superconductor in contact with two normal leads were predicted to be a signature of Majorana edge states [42,43], in the regime where the applied voltage is smaller than the energy splitting δE_M between them. (By contrast, the cross-correlations vanish in the limit $\delta E_M \to 0$ [44].) Recently, positive cross-correlations were measured in hybrid structures with tunnel junctions [45] and semiconducting nanowires [46].

Motivated by these results, one of us studied the cross-correlations of multiple Andreev reflections in a normal chaotic dot attached to three superconducting leads [47]. Depending on the coupling parameters, it was found that the cross-correlations acquire the same amplification factor as the shot noise. Furthermore, a sign reversal at low voltage was predicted under certain conditions. However, this study was restricted to the incoherent regime, when the junction does not carry a supercurrent. This regime may occur when a small magnetic flux is applied to the junction to suppress the Josephson coupling between the leads. An incoherent regime is also expected at a temperature or an applied bias smaller than the gap, but larger than the energy scale that would characterize the induced minigap in the density of states of the dot in equilibrium. Cross-correlations in junctions with three superconducting terminals were measured recently [48]. However, in that experiment only negative cross-correlations were observed.

The present work addresses the complementary coherent regime, where both an a.c. Josephson effect and dissipative quasiparticle transport take place. Phase-dependent multiple Andreev reflections in the $I(V)$-characteristics were investigated experimentally in a diffusive conductor [49] and studied theoretically both in a single mode [50] and a diffusive [51] junction. Our aim is to demonstrate that large positive cross-correlations may also occur in the coherent regime. The outline of the paper is the following: in Section 2 we introduce the scattering theory of multiple Andreev reflections. Then we present the results for the current in Section 3 and for the noise and cross-correlations in Section 4. Section 5 contains the conclusions and outlook.

2. Scattering theory of multiparticle Andreev reflection

We consider a junction consisting of a normal scattering region connected to three superconducting leads, see Fig. 1. Two leads (with labels α=1,2) are grounded, while the third lead (α=3) is biased with the voltage V. Furthermore, the superconducting loop geometry between leads 1 and 2 allows for imposing a superconducting phase difference ϕ that is tunable with the application of a magnetic flux through the loop.

For simplicity, we assume that there is one channel per terminal, that the normal scattering region does not break time-reversal symmetry, and that there is no spin–orbit coupling in the system. Then the normal region can be characterized by a 3×3 scattering matrix, $\hat{S}(\varepsilon) = \{s_{\alpha\gamma}(\varepsilon)\}_{\alpha,\gamma=1,2,3}$. If it is shorter than the superconducting coherence length, the energy-dependence of $\hat{S}(\varepsilon)$ can be neglected. Assuming that leads 1 and 2 are symmetrically coupled to lead 3, the scattering matrix can be parametrized as

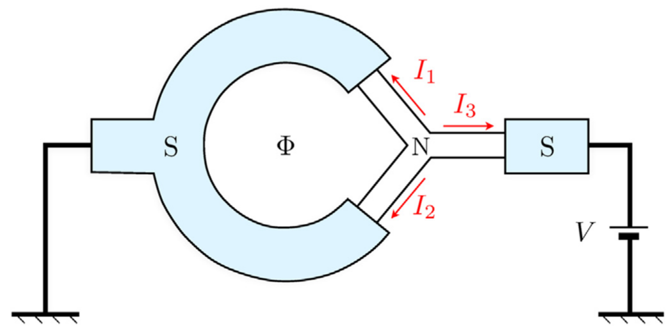

Fig. 1. Setup of a three-terminal superconducting junction. The superconducting leads are connected through a normal scattering region. A magnetic flux Φ applied through the loop formed between leads 1 and 2 allows controlling their superconducting phase difference, $\phi = 2\pi\Phi/\Phi_0$, where $\Phi_0 = hc/(2e)$ is the superconducting flux quantum. A voltage bias V is applied to lead 3 while the leads 1 and 2 are grounded.

$$\hat{S} = \begin{pmatrix} \sqrt{R}e^{ia} & \sqrt{D} & \sqrt{D_2} \\ \sqrt{D} & \sqrt{R}e^{ia} & \sqrt{D_2} \\ \sqrt{D_2} & \sqrt{D_2} & \sqrt{R_2}e^{ib} \end{pmatrix}, \tag{1}$$

up to irrelevant phases. Here $R = 1 - D - D_2$, $R_2 = 1 - 2D_2$, $a = \arccos[-D_2/(2\sqrt{RD})]$, and $b = \arccos[(D_2 - 2D)/(2\sqrt{R_2 D})]$. The scattering matrix is thus parametrized by two real parameters: the transparency D between leads 1 and 2 and the transparency $2D_2$ between lead 3 and the other leads, with the constraints $D \le 1$ and $D_2 \le 2\sqrt{D}(1-\sqrt{D})$.

When ϕ=0, the junction is equivalent to a two-terminal junction with transparency $2D_2$. However, the symmetry between leads 1 and 2 is broken at finite ϕ. This can be related to the formation of a doubly-degenerate Andreev bound state with energy

$$E_A(\varphi_1, \varphi_2, \varphi_3) = \Delta\sqrt{1 - D\sin^2\frac{\varphi_1-\varphi_2}{2} - D_2\left(\sin^2\frac{\varphi_2-\varphi_3}{2} + \sin^2\frac{\varphi_3-\varphi_1}{2}\right)}, \tag{2}$$

when the junction is in equilibrium [52,53]. Here φ_α is the superconducting phase of terminal α and $\phi = \varphi_1 - \varphi_2$. For $D_2 \ll 1$ and finite voltage, a quasi-bound state with energy $E_{qb}(\phi) = \Delta\sqrt{1 - D\sin^2(\phi/2)}$ between leads 1 and 2 remains and affects the current as well as the noise and cross-correlations. The phase dependence of I_3, where I_α is the current going to contact α = 1, 2, 3, was studied in Ref. [50]. Below we extend their results by computing the effect of multiple Andreev reflections on the current flowing between leads 1 and 2, $(I_1 - I_2)/2$, as well as the current noise, S_{33}, and the cross-correlations, S_{12}.

To calculate the transport properties of the junction, we make use of the Landauer–Büttiker theory, extended to describe hybrid junctions with superconducting leads [4,5]. For this, we first derive the wavefunctions associated with scattering states, which solve the time-dependent Bogoliubov–de Gennes equations describing the junction. In particular, the incoming and outgoing wavefunctions associated with an incoming electron-like state from lead β at energy E can be decomposed into their electron (e) and hole (h) amplitudes on the normal side of the interface between the junction and lead α,

$$\hat{\psi}^{\text{in/out}}_{e\beta E}(t) = \left\{\psi^{\text{in/out,e}}_{e\beta E,1}(t), \psi^{\text{in/out,e}}_{e\beta E,2}(t), \psi^{\text{in/out,e}}_{e\beta E,3}(t), \psi^{\text{in/out,h}}_{e\beta E,1}(t), \psi^{\text{in/out,h}}_{e\beta E,2}(t), \psi^{\text{in/out,h}}_{e\beta E,3}(t)\right\}^T \tag{3}$$

Using their Fourier transform in energy space, $\hat{\psi}^{\text{in/out}}_{e\beta E}(t) = \int d\varepsilon/(2\pi)\hat{\psi}^{\text{in/out}}_{e\beta E}(\varepsilon)e^{-i\varepsilon t}$, we can relate the incoming and outgoing components of the electron part of the wave function through the scattering matrix $\hat{S}^e$ for electrons. Namely,