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Reprint of : Thermodynamic properties of a quantum Hall anti-dot interferometer

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HIGHLIGHTS

- Quantum Hall interferometers based on a quantum anti-dot geometry.
- Thermodynamic considerations used to reflect interference transport phenomena.
- Analysis of the effect of electro-static Coulomb interactions between edge modes.
- Analysis of two regimes: Aharonov Bohm (AB) and Coulomb dominated regimes.
- Anti-dot compared with dot interferometers: easier access of the AB regime.
- Discussion of the relevance to recent measurements on anti-dot interferometers.

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ABSTRACT

We study quantum Hall interferometers in which the interference loop encircles a quantum anti-dot. We base our study on thermodynamic considerations, which we believe reflect the essential aspects of interference transport phenomena. We find that similar to the more conventional Fabry–Perot quantum Hall interferometers, in which the interference loop forms a quantum dot, the anti-dot interferometer is affected by the electro-static Coulomb interaction between the edge modes defining the loop. We show that in the Aharonov–Bohm regime, in which effects of fractional statistics should be visible, is easier to access in interferometers based on anti-dots than in those based on dots. We discuss the relevance of our results to recent measurements on anti-dots interferometers.

1. Introduction

Early after the discovery of the fractional quantum Hall states it was realized that the charged excitations that characterize these states carry fractional charge and satisfy fractional statistics, abelian or non-abelian. Abelian fractional statistics is manifested in the phase accumulated by one quasi-particle going around another [1–4]. The natural arena for an experimental observation of such a phase is that of interferometry. Consequently, large experimental and theoretical effort has been devoted to the study of quantum Hall interferometers of various types, such as the Fabry–Perot [5] and the Mach–Zehnder [6] interferometers. In these interferometers, current is introduced from a source and is distributed between two drains. The relative distribution between the drains involves interference between trajectories that go around an interference loop.

In an idealized model, when an integer quantum Hall state is examined, a variation of the magnetic field continuously varies the flux in the interference loop thus leading to Aharonov–Bohm oscillations. In fractional quantum Hall states another factor becomes important: the variation of the flux affects the number of localized quasi-particles within the loop. Since at low temperature this number is quantized to an integer, and since the mutual statistics of the interfering quasi-particle with the localized one is fractional, for abelian quantum Hall states the introduction of a localized quasi-particle into the bulk leads to a phase jump. As the temperature is raised the phase jump is expected to gradually smear.

A major deviation from the idealized model occurs in a Fabry–Perot interferometer due to the capacitive coupling between the interferometer's edge and bulk. As a result of this coupling, the variation of the magnetic field varies also the area of the interferometer, thus complicating the dependence of the Aharonov–Bohm phase on

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the field [7]. Furthermore, the introduction of a quasi-particle into the loop leads also to a sharp change in the area, affecting the discontinuous jump in the phase. The bulk-edge capacitive coupling was analyzed in details in [8], where a distinction has been made between the Aharonov–Bohm (AB) case, where the capacitive coupling is weak and the idealized model is a good approximation, and the Coulomb-dominated (CD) case, where the capacitive coupling is strong and the idealized model does not hold. This distinction holds both in the limit in which the interferometer is fairly open, and only two trajectories interfere, and in the limit of a closed interferometer (a quantum dot), in which multiple reflections are important, and the sinusoidal interference patterns are replaced by resonances.

In this work we examine interference in systems based on an anti-dot (AD) embedded in a bulk that is in a quantum Hall state [11–25]. A typical set-up is depicted in Fig. 1.[1] The anti-dot is a region where the density is fully depleted. As a consequence, it is encircled by edge modes of finite size. When the antidot is weakly coupled to the two ends of a quantum point contact, it induces back-scattering of the current in a quantum Hall device, provided that it is not blockaded by Coulomb charging energy. The AD may be tuned between transmission resonance peaks and dips by means of a magnetic field or gate voltage. We explore the dependence of the Coulomb blockade peaks on the magnetic field and on a gate voltage applied to the antidot, and analyze the information contained in this dependence on the mutual statistics of quasi-particles. We find that, when compared to a Fabri–Perot interferometer, the antidot geometry is potentially much easier to drive into the AB regime in which this information is accessible. Although the analysis we carry out is of thermodynamic quantities, namely the equilibrium charge on the anti-dot, we expect its essential features to be reflected also in transport, due to the coupling between transport and thermodynamics in the Coulomb blockade regime.

2. The model

In the case of weak coupling of the AD to the quantum point contacts (Fig. 1(a)), the charge on the anti-dot is approximately quantized in units of the charged excitations of the quantum Hall state that surrounds the antidot. Resonant backscattering through the edge modes encircling the AD takes place only when there is a degeneracy of the ground state energy of the antidot for two values of the charge. To specify that energy we define a model for the antidot. We focus on the case of two edge modes surrounding the antidot, denoted by 1 and 2 (inner and outer edges, respectively). The filling factor at the constriction is denoted by ν_2. The antidot, being depleted, is at filling factor zero. The filling factor between the two edge modes is denoted by ν_1. The outer edge channel separates two quantized Hall states corresponding to rational filling factors ν_1 and ν_2. The states ν_1 and ν_2 are either integer quantized Hall (IQH) states, or fractional quantum Hall (FQH) states described in the composite fermions picture. The corresponding quasi-particle excitations are given by e_1 and e_2. In the $T=0$ limit, the total charge in the AD island is quantized in units of e_2.

We first assume that there is no tunneling of charge between the two edge modes around the antidot, so that charge is quantized in each of the modes. We denote by N the total number of quasi-particles on the two closed edge channels, and by N_1 the number of quasi-particles on the inner edge channel. Since N and N_1 depend only on what happens on the two encircling edge channels, we define an energy functional, $E(N, N_1)$, to be the total energy of the system when N and N_1 are specified, minimized with respect to all other dynamical variables. In the absence of tunneling between the two edge modes we can approximate the energy of the modes by a quadratic form in terms of N, N_1

$$E(N, N_1) = \frac{K_1}{2}\left(\delta n_1\right)^2 + \frac{K_2}{2}\left(\delta n_2\right)^2 + K_{12}\delta n_1 \delta n_2 \tag{1}$$

where δn_i $(i=1,2)$ is the deviation of the charge on the ith interfering AD trapped edge channel in units of the electron charge e, and given by

$$\delta n_1 = N_1 e_1 + \phi_1 \Delta\nu_1 - \bar{q}_1 \tag{2}$$

$$\delta n_2 = (Ne_2 - N_1 e_1) + \phi_2 \Delta\nu_2 - \bar{q}_2 \tag{3}$$

The flux through the closed interfering edges is given by $\phi_i = \frac{BA_i}{\phi_0}$, with A_i being the area enclosed by the ith interfering edge. The terms $\phi_i \Delta\nu_i$ (with $i=1,2$) are related to the quantized Hall conductance of the different incompressible regions. When the flux changes, the resulting deviation of charge on the edge is given by $\phi_i \Delta\nu$, with $\Delta\nu_1 = \nu_1 - 0$ and $\Delta\nu_2 = \nu_2 - \nu_1$. The quantities $\bar{q}_i$ describe the effect of the positive background on the equilibrium charge density distribution. The variation of $\bar{q}_i$ and the area A_i as V_g is being varied depends on the coupling of the gate to the interferometer, and is characterized (following [8]) by two parameters

$$\beta_i = \left(\frac{B}{\phi_0}\right)\frac{dA_i}{dV_g}, \quad \gamma_i = \frac{d\bar{q}_i}{dV_g} \tag{4}$$

We consider the case of an ideal side gate, where the effect of the gate voltage δV_g is to alter the location of the edge, without changing its density profile [10]. In the limit of a sharp density profile around the AD, γ_i is proportional to the distance between two adjacent edges (δR_i) while $\beta_i \bar{\nu}_i$ (where $\bar{\nu}_i \equiv \frac{n\phi_0}{B}$, n is the bulk electron density and $\bar{\nu}_i \geq \nu_i$ in the Coulomb blockade limit) is proportional to the radius R_i. Hence, for an AD configuration in the Coulomb blockade limit we assume that

$$\gamma_i \ll \beta_i \nu_i \tag{5}$$

The coefficients K_1 and K_2 are related to the capacitance of the respective edges to ground (denoted by C_1 and C_2), as described in detail in [7,8]. For the state to be stable, they must satisfy $K_1 K_2 > K_{12}^2$. For a steep density profile at the edge, we estimate: $A_2 = A_1 + \delta A$, leading to $C_1 = C_2 + \delta C$, where $\frac{\delta C}{C_1} \ll 1$, $\frac{\delta A}{A_2} \ll 1$, so that in general $K_1 \approx K_2$. The mutual coupling term K_{12} describes the coupling between the two edges. We consider the case where $K_{12} > 0$ such that there is a repulsive interaction between excess charges on the two edges. For brevity, we present here a calculation done under the assumption $K_1 = K_2$, and generalize to $K_1 \approx K_2$ in the figures. When $K_1 = K_2$ the macroscopic energy function is

$$E = \frac{K_A}{2}(\delta n_1 + \delta n_2)^2 + \frac{K_B}{2}(\delta n_1 - \delta n_2)^2 \tag{6}$$

where $K_1 \approx K_2 = (K_A + K_B)$, $K_{12} = (K_A - K_B)$. Assuming that the mutual capacitance to be primarily electrostatic, the repulsive interaction between electrons leads to $K_A \geq K_B$. In this form the term $(\delta n_1 + \delta n_2)$ represents the total charge in the AD island, and the term involving $(\delta n_1 - \delta n_2)$ represents the energy cost associated with charge imbalance between the two modes encircling the AD. Since in the quantum Hall effect charge translates into distance, this term represents the dependence of the energy on the distance between the edge modes. A previous theoretical model [13] analyzed interference in ADs in a similar way, but dealt only with the limit where $K_B = 0$, and neglected the significant effect of nonzero K_B presented here.

[1] Ref. [8]'s terminology relates to ours as follows: $\nu_{in} = \nu_1$ and $\nu_{out} = \nu_2$.