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Reprint of : Flux sensitivity of quantum spin Hall rings

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HIGHLIGHTS

- We analyze the periodicity of persistent currents in quantum spin Hall (QSH) loops.
- One loop is partly covered by a superconductor.
- Time-reversal symmetry and parity conservation can constrain the period.
- A period of twice the quantum of flux is characteristic of QSH insulators.

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ABSTRACT

We analyze the periodicity of persistent currents in quantum spin Hall loops, partly covered with an *s*-wave superconductor, in the presence of a flux tube. Much like in normal (non-helical) metals, the periodicity of the single-particle spectrum goes from $\Phi_0 = h/e$ to $\Phi_0/2$ as the length of the superconductor is increased past the coherence length of the superconductor. We further analyze the periodicity of the persistent current, which is a many-body effect. Interestingly, time reversal symmetry and parity conservation can significantly change the period. We find a $2\Phi_0$-periodic persistent current in two distinct regimes, where one corresponds to a Josephson junction and the other one to an Aharonov–Bohm setup.

1. Introduction

In a seminal work from 1986, Büttiker and Klapwijk discussed the flux sensitivity of a piecewise normal and superconducting metal loop [1], as showed in Fig. 1(a). The model they considered in order to describe such a system – a single electronic channel with a linearized spectrum – is as simple as it gets, yet captures the characteristic features associated with persistent currents in mesoscopic loops. Indeed, in the Andreev approximation, and in the low-energy regime, microscopic details of the model hardly matter. As long as the length of the normal region is much larger than the coherence length of the superconductor, the persistent current will have the familiar saw tooth shape, both in the normal and superconducting regime. What changes though between the two regimes is the periodicity of the superconducting current with the applied flux. Following a simple calculation of the excitation spectrum, Büttiker and Klapwijk were able to show how as the length of the superconducting region is progressively increased, the periodicity of the persistent current is halved, going from Φ_0 to $\Phi_0/2$, with $\Phi_0 = h/e$ being the quantum of flux. The almost thirty years since the 1986 paper have seen many exciting discoveries in the field of mesoscopic physics. One of them is the advent of topological insulators [2–5]. Of particular interest to us here is the case of quantum spin Hall insulators and their one-dimensional helical edge states, for several reasons. They first offer a new realization of 1D Dirac physics, that goes beyond linearization of quadratic spectra at the Fermi points. Second, helicity, that is the locking of direction of spin with the direction of motion, protects transport against time-reversal invariant impurities. More precisely, single-particle elastic backscattering is forbidden by time-reversal symmetry, leading the community in a vast effort to better understand the effect of inelastic scattering in these systems [6–16]. Third, the interplay of helicity and superconductivity imposes a constraint on the parity of the number of quasi-particles, or fermion parity (FP), in SNS junctions based on helical liquids, as the one depicted in Fig. 1 (b). This results in a so-called fractional Josephson effect [17–19], with a Φ_0-periodic supercurrent. A few works have already discussed the physics of persistent currents in helical rings, highlighting the effects of magnetic and non-magnetic impurities [20], or the hybridization between edge states in narrow quantum spin Hall rings [21]. In the present paper, we revisit the analysis of Büttiker and Klapwijk in the context of quantum spin Hall insulators. In particular we analyze the crossover between the normal

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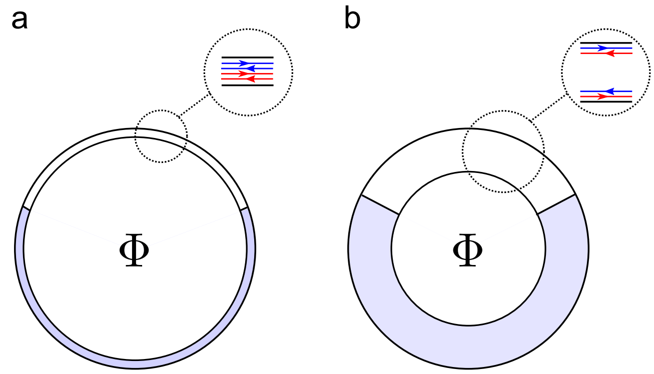

Fig. 1. (a) Original setup as proposed in Ref. [1]. A piecewise normal and superconducting loop of size L comprising a single conducting channel is threaded by a magnetic flux Φ. Close to the Fermi points, there are four available modes: spin up or spin down right movers (blue solid lines) and spin up or spin down left movers (red solid lines). The normal region has length d_N while the superconducting region has length d_S. (b) A quantum spin Hall loop, partly covered by an s-wave superconductor (shaded region) and threaded by a flux Φ. Each edge hosts half the degrees of freedom available in setup (a), resulting in two helical channels physically separated by the insulating bulk. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

persistent current and the supercurrent, as the length of the superconducting region is increased, and discuss their periodicity. We argue that a constraint from time reversal symmetry doubles the period in the normal case, as compared to Ref. [1]. A similar effect is already known to occur in the superconducting case, and was put forward by Zhang and Kane in Ref. [22].

The outline of the paper is as follows. In Section 2, we start by discussing the model and the various symmetries that constrain the periodicity at a given edge, both in the normal and superconducting regimes, and contrast the results with the case of a non-helical metal. Then, in Section 3, we comment on the total persistent current, when both edges are taken into account. Finally, in Section 4, we give some conclusions.

2. Model and persistent current at a given edge

2.1. Normal helical ring

2.1.1. Single-particle spectrum

We are interested in modeling the setup of Fig. 1(b). To that end, we first restrict our analysis to a single edge, say the outer one. Before following in the footsteps of Ref. [1] and computing the excitation spectrum in the superconducting case, let us consider first the normal case. We model the outer edge by a 1D segment of size L along the x direction and impose periodic boundary conditions. The flux is included via the minimal substitution of the momentum operator $\hat{p}_x = -i\hbar\partial_x$,

$$\hat{p}_x \to \hat{p}_x - qA \tag{1}$$

with q being the charge of the particles and $A = \Phi/L$ a vector potential. In the following we take $q = -e$, for electrons. At energies much smaller than the bulk band gap, the helical states are well described by Dirac fermions, with the following single particle Hamiltonian:

$$\mathcal{H} = v_F(\hat{p}_x + eA)\sigma_3, \tag{2}$$

where v_F is the Fermi velocity and σ_3 is the usual Pauli matrix, acting on spin space. Right moving electrons have therefore spin up while left moving electrons have spin down (the situation is reversed at the inner edge). The single particle spectrum consists of two branches

$$\varepsilon_{\pm,n}(\Phi) = \pm \hbar v_F \frac{2\pi}{L}\left(n + \frac{\Phi}{\Phi_0}\right), \tag{3}$$

with $\Phi_0 = h/e$. Corresponding eigenstates are of the form

$$\phi_{\pm,n}(x) = \chi_\pm e^{ik_n x}/\sqrt{L}, \tag{4}$$

with $\chi_+ = (1,0)^T$, $\chi_- = (0,1)^T$ and the momentum $k_n = 2\pi n/L$ is quantized due to periodic boundary conditions.

2.1.2. Excitations

In the absence of the flux, $\Phi=0$, we take the chemical potential to be at the Dirac point, that is, all states with negative energy are filled. Given the Hamiltonian of Eq. (2), nothing indicates that the spectrum is bounded from below, which would mean the ground state energy is infinite. Of course, in a real system, there is a natural cutoff scale, as the spectrum is bounded both from below and above by the bulk bands. However, at this stage one can let the cutoff go to infinity and renormalize the ground state energy without affecting the general physics which is given by the low energy excitations with respect to the Fermi sea. We then define the ground state $|0\rangle_0$ such that

$$c^\dagger_{\pm,n}|0\rangle_0 = 0, \quad n \le 0, \tag{5}$$

$$c_{\pm,n}|0\rangle_0 = 0, \quad n > 0 \tag{6}$$

and we impose that $|0\rangle_0$ has zero energy. The index 0 here serves as a reminder that the state is defined for $\Phi=0$. There are two types of excitations on top of $|0\rangle_0$. One can create a particle in the conduction band, for instance $c^\dagger_{\pm,n>0}|0\rangle_0$, or create a hole in the valence band, for instance $c_{\pm,n\le 0}|0\rangle_0$. Importantly, particle and hole excitations are independent. We can also define a many-body Hamiltonian (still for $\Phi=0$) as

$$H = \int_0^L dx\, :\Psi^\dagger(x)\mathcal{H}\Psi(x):\,, \tag{7}$$

with $\Psi(x) = [\psi_+(x), \psi_-(x)]^T$ being a quantum field and where $:\ldots:$ indicates normal-ordering, ensuring that indeed ${}_0\langle 0|H|0\rangle_0 = 0$. Among the many possible excited states, some will play a particular role in the following. These are the states with a finite number of particles, but no particle–hole excitations. They are obtained by filling positive energy states or emptying negative energy states in the following way. The state with $N_{j=\pm}$ particles is defined by

$$|N_j\rangle_0 = \prod_{n=1}^{N_j} c^\dagger_{j,n}|0\rangle_0 \quad \text{if } N_j > 0, \tag{8}$$

$$|N_j\rangle_0 = \prod_{n=0}^{N_j-1} c_{j,-n}|0\rangle_0 \quad \text{if } N_j < 0. \tag{9}$$

In the following we use the notation $|N_+, N_-\rangle_0$ in order to refer to a state with N_+ right movers and N_- left movers. The energy of these many-body states is simply given by [23]

$$E(N_+, N_-, \Phi = 0) = \hbar v_F \frac{2\pi}{L}\frac{1}{2}\sum_{j=\pm} N_j(N_j + 1). \tag{10}$$

This prompts us to introduce the so-called chiral current operators

$$J_\pm(x) = \,:\psi^\dagger_\pm(x)\psi_\pm(x):\,, \tag{11}$$

as well as the particle current operator

$$J(x) = v_F :\Psi^\dagger(x)\sigma_3\Psi(x):\,. \tag{12}$$