Physica E 82 (2016) 247–253

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Reprint of : Nanomagnet coupled to quantum spin Hall edge: An adiabatic quantum motor

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HIGHLIGHTS

- The study identifies topological insulator edge coupled to nanomagnet as adiabatic quantum motor.
- Landauer–Büttiker theory for Landau–Lifshitz–Gilbert equation of nanomagnet dynamics.
- The study shows that this system realizes a Thouless motor and discusses its efficiency.

ARTICLE INFO

Article history:
Received 15 July 2015
Received in revised form
16 August 2015
Accepted 18 August 2015
Available online 14 March 2016

Keywords:
Quantum spin Hall effect
Topological insulators
Magnetization dynamics

ABSTRACT

The precessing magnetization of a magnetic islands coupled to a quantum spin Hall edge pumps charge along the edge. Conversely, a bias voltage applied to the edge makes the magnetization precess. We point out that this device realizes an adiabatic quantum motor and discuss the efficiency of its operation based on a scattering matrix approach akin to Landauer–Büttiker theory. Scattering theory provides a microscopic derivation of the Landau–Lifshitz–Gilbert equation for the magnetization dynamics of the device, including spin-transfer torque, Gilbert damping, and Langevin torque. We find that the device can be viewed as a Thouless motor, attaining unit efficiency when the chemical potential of the edge states falls into the magnetization-induced gap. For more general parameters, we characterize the device by means of a figure of merit analogous to the ZT value in thermoelectrics.

1. Introduction

Following Ref. [1], Meng et al. [2] recently showed that a transport current flowing along a quantum spin Hall edge causes a precession of the magnetization of a magnetic island which locally gaps out the edge modes (see Fig. 1 for a sketch of the device). The magnetization dynamics is driven by the spin transfer torque exerted on the magnetic island by electrons backscattering from the gapped region. Indeed, the helical nature of the edge state implies that the backscattering electrons reverse their spin polarization, with the change in angular momentum transferred to the magnetic island. This effect is not only interesting in its own right, but may also have applications in spintronics.

Current-driven directed motion at the nanoscale has also been studied for mechanical degrees of freedom, as motivated by progress on nanoelectromechanical systems. Qi and Zhang [3] proposed that a conducting helical molecule placed in a homogeneous electrical field could be made to rotate around its axis by a transport current and pointed out the intimate relations with the concept of a Thouless pump [4]. Bustos-Marun et al. [5] developed a general theory of such adiabatic quantum motors, used it to discuss their efficiency, and emphasized that the Thouless motor discussed by Qi and Zhang is optimally efficient.

It is the purpose of the present paper to emphasize that the current-driven magnetization dynamics is another – perhaps more experimentally feasible – variant of a Thouless motor and that the theory previously developed for adiabatic quantum motors [5] is readily extended to this device. This theory not only provides a microscopic derivation of the Landau–Lifshitz–Gilbert equation for the current-driven magnetization dynamics, but also allows one to discuss the efficiency of the device and to make the relation with the magnetization-driven quantum pumping of charge more explicit.

Specifically, we will employ an extension of the Landauer–Büttiker theory of quantum transport which includes the forces exerted by the

DOI of original article: http://dx.doi.org/10.1016/j.physe.2015.08.031
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http://dx.doi.org/10.1016/j.physe.2016.02.037
1386-9477/© 2016 Published by Elsevier B.V.

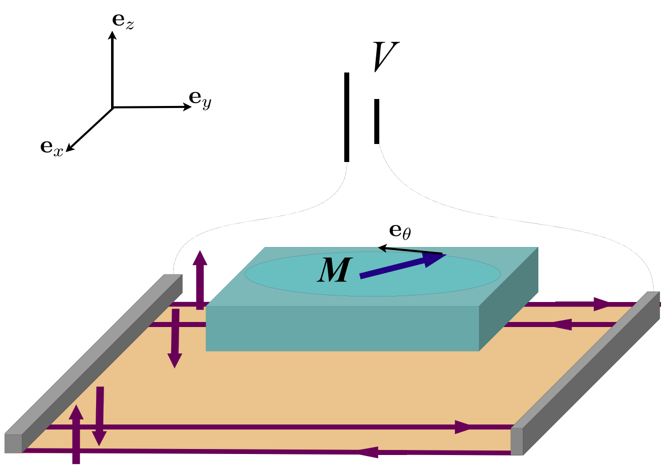

Fig. 1. Schematic setup. A nanomagnet with magnetic moment **M** couples to a Kramers pair of edge states of a quantum spin Hall insulator. The effective spin current produces a spin-transfer toque and the magnetic moment precesses.

electrons on a slow classical degree of freedom [6–9]. Markus Büttiker developed Landauer's vision of quantum coherent transport as a scattering problem into a theoretical framework [10,11] and applied this scattering theory of quantum transport to an impressive variety of phenomena. These applications include Aharonov–Bohm oscillations [12], shot noise and current correlations [11,13,14], as well as edge-state transport in the integer Hall effect [15] and topological insulators [16]. Frequently, Büttiker's predictions based on scattering theory provided reference points with which other theories – such as the Keldysh Green-function formalism [17–20] or master equations [21] – sought to make contact.

In the present context, it is essential that scattering theory also provides a natural framework to study quantum coherent transport in systems under time-dependent driving. For adiabatic driving, Büttiker's work with Thomas and Prêtre [22] was instrumental in developing a description of adiabatic quantum pumping [4] in terms of scattering theory [23–26] which provided a useful backdrop for later experiments [27–31]. Beyond the adiabatic regime, Moskalets and Büttiker combined the scattering approach with Floquet theory to account for periodic driving [32]. These works describe adiabatic quantum transport as a limit of the more general problem of periodic driving and ultimately triggered numerous studies on single-particle emitters and quantum capacitors (as reviewed by Moskalets and Haack in this volume [33]).

The basic idea of the adiabatic quantum motor [5] is easily introduced by analogy with the Archimedes screw, a device consisting of a screw inside a pipe. By turning the screw, water can be pumped against gravity. This is a classical analog of a quantum pump in which electrons are pumped between reservoirs by applying periodic potentials to a central scattering region. Just as the Archimedes pump can pump water against gravity, charge can be quantum pumped against a voltage. In addition, the Archimedes screw has an inverse mode of operation as a *motor*: water pushed through the device will cause the screw to rotate. The adiabatic quantum motor is a quantum analog of this mode of operation in which a transport current pushed through a quantum coherent conductor induces unidirectional motion of a classical degree of freedom such as the rotations of a helical molecule.

The theory of adiabatic quantum motors [5,34] exploits the assumption that the motor degrees of freedom – be they mechanical or magnetic – are slow compared to the electronic degrees of freedom. In this adiabatic regime, the typical time scale of the mechanical dynamics is large compared to the dwell time of the electrons in the interaction region between motor and electrical degrees of freedom. In this limit, the dynamics of the two degrees of freedom can be discussed in a mixed quantum-classical description. The motor dynamics is described in terms of a classical equation of motion, while a fully quantum-coherent description is required for the fast electronic degrees of freedom.

From the point of view of the electrons, the motor degrees of freedom act as *ac* potentials which pump charge through the conductor. Conversely, the backaction of the electronic degrees of freedom enters through adiabatic reaction forces on the motor degrees of freedom [6–9]. When there is just a single (Cartesian) classical degree of freedom, these reaction forces are necessarily conservative, akin to the Born–Oppenheimer force in molecular physics [35]. Motor action driven by transport currents can occur when there is more than one motor degree of freedom (or a single angle degree of freedom). In this case, the adiabatic reaction force need no longer be conservative when the electronic conductor is subject to a bias voltage [6–9].

In next order in the adiabatic approximation, the electronic system also induces frictional and Lorentz-like forces, both of which are linear in the slow velocity of the motor degree of freedom. Including the fluctuating Langevin force which accompanies friction yields a classical Langevin equation for the motor degree of freedom. This equation can be derived systematically within the Keldysh formalism [35] and the adiabatic reaction forces expressed through the scattering matrix of the coherent conductor [6–8].

While these developments focused on mechanical degrees of freedom, it was also pointed out that the scattering theory of adiabatic reaction forces extends to magnetic degrees of freedom [9]. In this case, adiabaticity requires that the precessional time scale of the magnetic moment is larger than the electronic dwell time. The effective classical description for the magnetic moment takes the form of a Landau–Lifshitz–Gilbert (LLG) equation. Similar to nanoelectromechanical systems, the LLG equation can be derived systematically in the adiabatic limit for a given microscopic model and the coefficients entering the LLG equation can be expressed alternatively in terms of electronic Green functions or scattering matrices [36–39,9]. In the following, we will apply this general theory to a magnetic island coupled to a Kramers pair of helical edge states.

This work is organized as follows. Section 2 reviews the scattering-matrix expressions for the torques entering the LLG equation. Section 3 applies this theory to helical edge states coupled to a magnetic island and makes the relation to adiabatic quantum motors explicit. Section 4 defines and discusses the efficiency of this device and derives a direct relation between charge pumping and spin transfer torque. Section 5 is devoted to conclusions.

2. S-matrix theory of spin transfer torques and Gilbert damping

2.1. Landau–Lifshitz–Gilbert equation

Consider a coherent (Landauer–Büttiker) conductor coupled to a magnetic moment. The latter is assumed to be sufficiently large to justify a classical description of its dynamics but sufficiently small so that we can treat it as a single macrospin. Then, its dynamics is ruled by a Landau–Lifshitz–Gilbert equation

$$\dot{\mathbf{M}} = \mathbf{M} \times \left[-\partial_{\mathbf{M}} U + \mathbf{B}_{\rm el} + \delta \mathbf{B} \right]. \qquad (1)$$

Note that we use units in which **M** is an angular momentum and for simplicity of notation, $\mathbf{B}_{\rm el}$ as well as $\delta\mathbf{B}$ differ from a conventional magnetic field by a factor of g_d, the gyromagnetic ratio of the macrospin. The first term on the right-hand side describes the dynamics of the macrospin in the absence of coupling to the electrons. It is derived from the quantum Hamiltonian