Physica E 82 (2016) 310–313

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Reprint of : Quantum point contacts as heat engines

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HIGHLIGHTS

- We consider a quantum point contact as a heat engine in nonequilibrium steady state.
- We determine the efficiency of the point contact at maximum power.
- Due to fluctuations, the efficiency may beat the Carnot limit on short time scales.

ARTICLE INFO

Article history:
Received 5 June 2015
Received in revised form
23 July 2015
Accepted 3 August 2015
Available online 14 March 2016

Keywords:
Mesoscopic physics
Quantum transport
Noise and fluctuations
Quantum thermodynamics

ABSTRACT

The efficiency of macroscopic heat engines is restricted by the second law of thermodynamics. They can reach at most the efficiency of a Carnot engine. In contrast, heat currents in mesoscopic heat engines show fluctuations. Thus, there is a small probability that a mesoscopic heat engine exceeds Carnot's maximum value during a short measurement time. We illustrate this effect using a quantum point contact as a heat engine. When a temperature difference is applied to a quantum point contact, the system may be utilized as a source of electrical power under steady state conditions. We first discuss the optimal working point of such a heat engine that maximizes the generated electrical power and subsequently calculate the statistics for deviations of the efficiency from its most likely value. We find that deviations surpassing the Carnot limit are possible, but unlikely.

Büttiker was among the first scientists to realize that measurements of current fluctuations deliver most valuable information about the internal structure of mesoscopic conductors [1]. The measurement of shot noise [2] in a tunnel junction, for instance, may be used to determine the elementary charge of the charge carriers transferred through the circuit. Its measurement may as well serve to reveal the transmission probabilities of a multichannel mesoscopic point contact.

The description of current fluctuations was later extended to full statistics of the charge transfer through a mesoscopic conductor [3]. From an experimental point of view, current fluctuations are probably the easiest to measure. Nevertheless, statistics for a number of other mesoscopic physical quantities have also been investigated: among them, combined charge-phase statistics in the superconducting state [4], waiting time statistics of a closed volume [5], voltage statistics on a current biased point contact [6].

Recently, interest has shifted to energy transport through mesoscopic structures. The study of energy transport is partially motivated by the possibility to use small circuits to convert local temperature differences into voltages [7]. Heat currents are subject to fluctuations as well [8]. Such fluctuations have been theoretically studied in a number of situations [9–13]. Although the direct measurement of fluctuations in heat current is probably difficult, indirect consequences of energy fluctuations have been observed experimentally [14–16].

In this paper, we apply the theory of heat current fluctuations to a question of rather conceptual than practical interest. We consider a mesoscopic heat engine that converts heat partially into electrical work. Since heat currents in mesoscopic devices fluctuate with time, any quantity derived from the heat currents will fluctuate as well. In particular, the efficiency of heat to work conversion will depend randomly on time and therefore may exceed the Carnot efficiency for a short time, not on average, but sometimes with a non-vanishing probability. It is the aim of this work to quantify this probability. We illustrate our discussion with a quantum point contact, a narrow constriction between two electrodes that shows quantized linear conductance.

1. System and formalism

Our system of interest is a mesoscopic point contact coupling

DOI of original article: http://dx.doi.org/10.1016/j.physe.2015.08.003
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http://dx.doi.org/10.1016/j.physe.2016.02.045
1386-9477/© 2016 Published by Elsevier B.V.

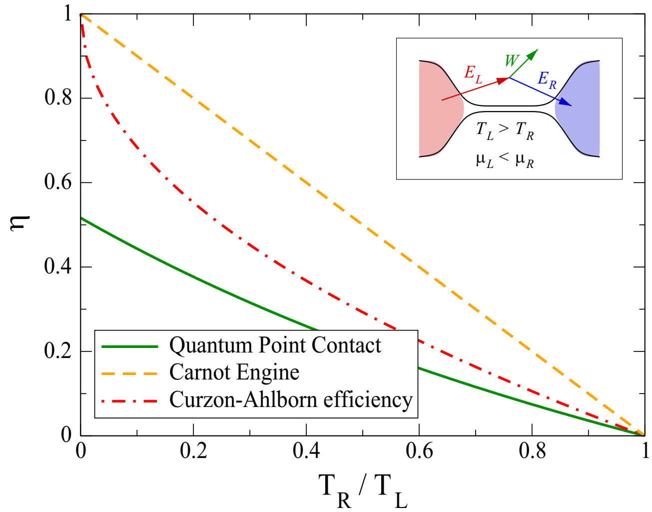

Fig. 1. *Inset:* A quantum point contact connected to two reservoirs at different temperatures T_L, T_R and chemical potentials μ_L, μ_R may be used as a heat engine that converts heat E_L partially into electrical work W. *Main plot:* Efficiency $\eta = W/E_L$ versus the applied temperature ratio T_R/T_L at the optimal working point, compared to the ideal efficiency of a Carnot process and the Curzon–Ahlborn efficiency at maximum power.

left (L) and right (R) reservoirs that have in general different chemical potentials μ_L, μ_R and different temperatures T_L, T_R. This is an experimentally relevant situation that has been the subject of recent works [17–19]. Quite generally, energy will flow from the left to the right reservoir when $T_L > T_R$. The energy flow is accompanied by a charge flow against a difference in the chemical potentials, $\mu_L < \mu_R$. This charge flow corresponds to the electrical work generated by the point contact. The inset of Fig. 1 shows the setup and the sign conventions. The heat extracted from the left reservoir E_L has a positive sign, the (smaller) heat evacuated into the right reservoir E_R has a negative sign. We take the generated work W as a positive quantity:

$$E_L + E_R = W > 0. \tag{1}$$

The efficiency of the heat engine is then defined as

$$\eta = \frac{W}{E_L} = \frac{E_L + E_R}{E_L}. \tag{2}$$

We consider that both electrodes have well defined chemical potentials and temperatures. Hence, relaxation processes in the reservoirs are assumed to be fast compared to the time τ that our measurement takes. At the end of the experiment we record the amount of heat E_L that is extracted from the hot reservoir and the amount of heat E_R that is dumped into the cold reservoir. Due to thermal and quantum fluctuations, the whole transfer process is probabilistic and described by a probability distribution $P_\tau(E_L, E_R)$. Often it is more useful to use the corresponding cumulant generating function $S_\tau(i\xi_L, i\xi_R)$ instead of P. Both quantities are linked by Fourier transformation:

$$P_\tau(E_L, E_R) \sim \int d\xi_L\, d\xi_R e^{-i(\xi_L E_L + \xi_R E_R) + S_\tau(i\xi_L, i\xi_R)}. \tag{3}$$

The generating function describing the statistics of heat transfer in a general two-terminal conductor is given in Ref. [8]. Here, we adapt this generating function for our purpose:

$$\begin{aligned} S_\tau(i\xi_L, i\xi_R) &= \frac{\tau}{\pi} \int d\varepsilon \ln\{1 + \Gamma f_L(1 - f_R)(e^{i\xi_L(\varepsilon-\mu_L) - i\xi_R(\varepsilon-\mu_R)} - 1) \\ &+ \Gamma(1 - f_L) f_R (e^{-i\xi_L(\varepsilon-\mu_L) + i\xi_R(\varepsilon-\mu_R)} - 1)\}. \end{aligned} \tag{4}$$

This expression applies to a spin-degenerate single-channel point contact with transparency Γ. It also contains the leads' Fermi occupation factors $f_{L,R} = 1/[1 + e^{(\varepsilon-\mu_{L,R})/T_{L,R}}]$. For convenience, we hereafter choose units such that the Planck constant, the unit charge and the Boltzmann constant are equal to one. Any cumulant of the distribution $P_\tau(E_L, E_R)$ can be obtained from the generating function by taking partial derivatives:

$$\langle\langle (E_L)^m (E_R)^n \rangle\rangle = \left. \frac{\partial^{m+n} S}{(\partial i\xi_L)^m (\partial i\xi_R)^n} \right|_{\xi_L = \xi_R = 0}. \tag{5}$$

Thermoelectric effects necessarily require that the transmission probability Γ of the quantum point contact depends on energy ε. A basic and convenient model for this dependence was proposed by Büttiker [20]. If the potential barrier creating the point contact is a saddle, the transmission probability of one single transmission channel reads

$$\Gamma(\varepsilon) = \frac{1}{1 + e^{-(\varepsilon - \varepsilon_0)/\omega_z}}, \tag{6}$$

where ε_0 is the potential at the saddle and ω_z gives the energy width of the transition region. In this work we will use a simplified version. We assume a very long point contact, $\omega_z \to \infty$, and choose the energy scale such that $\varepsilon_0 = 0$. The transmission probability then jumps sharply from zero to one

$$\Gamma(\varepsilon) = \begin{cases} 0, & \varepsilon < 0 \\ 1, & \varepsilon > 0, \end{cases} \tag{7}$$

when energy surpasses the threshold set by ε_0.

2. Optimal working point

For a given set of temperatures T_L and T_R we may optimize the chemical potentials μ_L and μ_R such that production of work is maximized on average. We call this optimized situation the working point of the point contact. The result is equivalent to that of the efficiency at maximum power which has been analyzed in detail in Ref. [21] in the context of scattering theory of quantum transport. From Eq. (1) we have

$$\langle\langle W \rangle\rangle = \langle\langle E_R \rangle\rangle + \langle\langle E_L \rangle\rangle, \qquad \frac{\partial \langle\langle W \rangle\rangle}{\partial \mu_{L,R}} = 0, \tag{8}$$

with the heat extracted from the heat reservoir given by

$$\langle\langle E_L \rangle\rangle \sim \tau \int_0^\infty d\varepsilon (f_L - f_R)(\varepsilon - \mu_L), \tag{9}$$

and the generated work obeying the Joule expression

$$\langle\langle W \rangle\rangle \sim \tau(\mu_R - \mu_L) \int_0^\infty d\varepsilon (f_L - f_R), \tag{10}$$

for the nonzero voltage difference $\mu_R - \mu_L$. Combining the derivatives with respect to both chemical potentials we find

$$\frac{\mu_L}{T_L} = \frac{\mu_R}{T_R}. \tag{11}$$

Since $T_L > T_R$ and $\mu_L < \mu_R$ (charge current must flow against the potential from the hot to the cold reservoir), it follows immediately that both chemical potentials have to be negative (we set the Fermi energy E_F=0). Substituting Eq. (11) into $\langle\langle W \rangle\rangle$ and calculating the energy integral yield the intermediate result