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Reprint of : Dynamics of a quantum wave emitted by a decaying and evanescent point source

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HIGHLIGHTS

- A point source boundary condition is used to model the dynamics in a tunnel region.
- The role of the Büttiker–Landauer time is emphasized.
- The model displays diffraction in time and deviations from the exponential decay.
- Optimal injection frequencies and detection positions are discussed.

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ABSTRACT

We put forward a model that describes a decaying and evanescent point source of non-interacting quantum waves in 1D. This point-source assumption allows for a simple description that captures the essential aspects of the dynamics of a wave traveling through a classically forbidden region without the need to specify the details of the inner region. The dynamics of the resulting wave is examined and several characteristic times are identified. One of them generalizes the tunneling time-scale introduced by Büttiker and Landauer and it characterizes the arrival of the maximum of the wave function. Diffraction in time and deviations from exponential decay are also studied. Here we show that there exists an optimal injection frequency and detection point for the observation of these two quantum phenomena.

1. Introduction

Since the pioneering work of Büttiker and Landauer (BL) [1,2], much effort has been devoted to understand and characterize the dynamics of wave functions representing the state of a quantum particle propagating through a tunneling region. At first the research was oriented to define a "tunnelling time" for the particle. Several candidates were put forward, apart from the BL time, and an intense debate followed [3,4]. It was later understood that since the projectors for locating the particle at the barrier and for finding the particle eventually transmitted do not commute, many possible quantizations of the classical concept of a traversal time through the barrier region are possible, and that several of them may be relevant depending on the experimental setting and/or quantity observed [5,4]. Thus, rather than seeking "the tunneling time", a research line aimed at describing the wave dynamics with a minimal number of elements emerged, which could include characteristic times for forerunners, main peaks, or transitions among different regimes [6–11].

Simplified models are instrumental in identifying the main phenomena and develop the necessary conceptual frame as well as a general theory. Among the different analytical models used to study transient phenomena in quantum mechanics [12], the "source models", where the wave function is given at a fixed position for all times, play a key role. They have been used to study diffraction in time [13–18], tunneling dynamics [7,9,10,19–23], dynamics on absorbing media [24,25], atom lasers [26], deviations from exponential decay [27], and the time of arrival [28–30]. The physical meaning of the source boundary conditions was clarified

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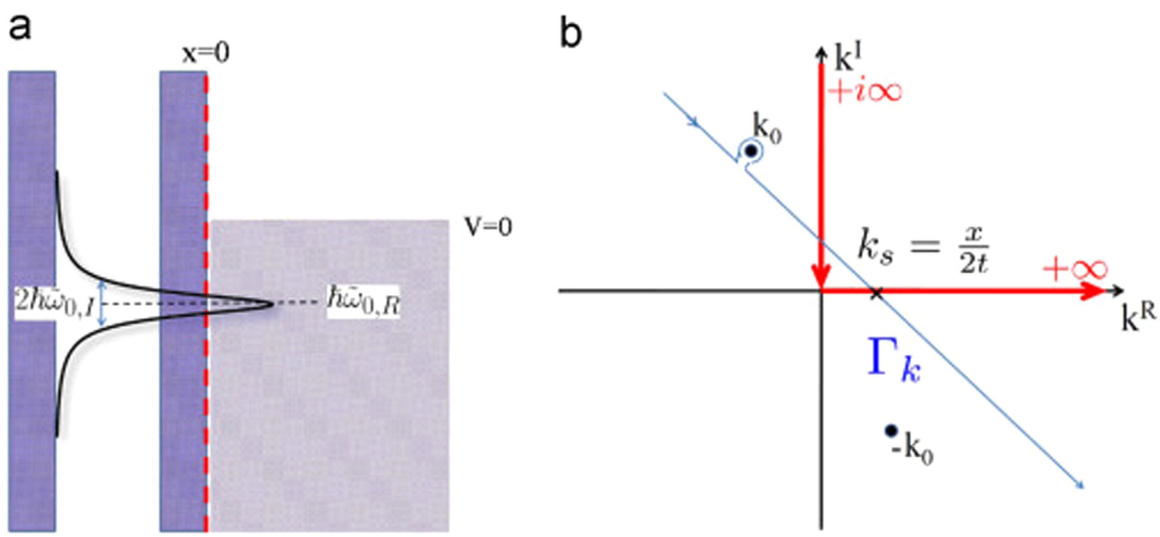

Fig. 1. Scheme of the model and details of the complex plane analysis. (a) The model accounts for the wave dynamics of a decaying resonance, with central frequency $\tilde{\omega}_{0,R} < 0$, into a tunnel region ($x > 0$) that induces an exponential depletion, with a characteristic lifetime $1/(2\tilde{\omega}_{0,I})$, of the initial state. (b) Original integration path γ in the complex k-space (red arrows) and contour Γ_k of integration for the w function crossing the $k_s = x/2t$ saddle point and passing above all possible poles (in our case, only the pole at k_0 may be crossed). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

in [30,17] by finding the connection between source boundary conditions and the more standard initial value problem.

In most applications of the point-source model, the emission (carrier) frequency was real, for either traveling or evanescent conditions (with real or imaginary wavenumber, respectively). An imaginary part was added to the carrier frequency in [27] to study deviations from exponential decay and their enhancement, and also in [18] to find a simple explanation of diffraction in time (DIT). In this work we consider a case so far overlooked, namely, a negative real part of the frequency corresponding to evanescent conditions and an imaginary part that produces decay. This completes the work of one of us with M. Büttiker in [7], which dealt with a purely evanescent source, without decay, and also fills the gap between this work and the decaying source considered in [27]. The physical setting corresponds to the 1D wave dynamics in an evanescent region (positions $x > 0$) for a decaying resonance which is depleted exponentially through some escape channel (say to the left) which is not represented explicitly in the model, see Fig. 1(a).

A surprising result for the purely evanescent source [7,32] was that a direct generalization of the BL time set a time scale for the wave density maximum in opaque (semiclassical) conditions, i.e., beyond the penetration length. This "forerunner", paradoxically, was not at all dominated by evanescent components but by a saddle point contribution above threshold. This finding provided a role for the BL timescale different from the ones that had been attributed so far to it (as a scale that determines the transition from sudden to adiabatic regimes for an oscillating barrier [1], and the rotation of the spin in a weak magnetic field in opaque conditions [2]). Here we shall generalize the BL time scale further for the decaying and evanescent source and specify its relation to the saddle-point dominated peak. Unlike [7], the decaying evanescent source allows for a power-law decay following the commonly observed exponential one (post-exponential regime), which we analyze in this work. As well, the modifications on DIT with respect to the decaying above-threshold source in [27] are examined.

2. Point source model

Let a source at the origin $\tilde{x} = 0$ be switched on suddenly at time $\tilde{t} = 0$. (Dimensional quantities wear a tilde here to distinguish them from dimensionless ones, without tilde.) The source *boundary* condition is

$$\tilde{\psi}(\tilde{x} = 0, \tilde{t}) = \Theta(\tilde{t})e^{-i\tilde{\omega}_0\tilde{t}}, \tag{1}$$

with complex (carrier) frequency $\tilde{\omega}_0 = \tilde{\omega}_0^R + i\tilde{\omega}_0^I$. The particle is assumed to move in one dimension and we consider the emission into $\tilde{x} \geq 0$. The initial wave function is 0 everywhere except at $\tilde{x} = 0$. This setting was studied in Refs. [18,27] assuming the propagating condition $\tilde{\omega}_0^R > 0$ and $\tilde{\omega}_0^I < 0$.

This corresponds to a simplified model to account for the propagation into the $\tilde{x} \geq 0$ region of an initially prepared resonant state with frequency $\tilde{\omega}_0^R$ above the cut-off frequency $\tilde{\omega}_c = 0$ and lifetime $-1/\tilde{\omega}_0^I$. Notice that, without loss of generality, we fix the constant potential V_0 in which the quantum particle moves as zero, and hence, frequencies below 0 lead to imaginary wavenumbers (evanescent waves).

By contrast, here we consider the evanescent injection below the media cut-off, i.e., $\tilde{\omega}_0^R \leq 0$.

In addition, since $|\tilde{\psi}(0, \tilde{t})|^2 = e^{2\tilde{\omega}_0^I t}$, we impose that $\tilde{\omega}_0^I \leq 0$ to model an exponentially decaying source.

The dispersion relation corresponding to the free particle (unlike [7], we set the constant potential level as V=0) is

$$\tilde{\omega}(\tilde{k}) = \frac{\hbar\tilde{k}^2}{2m}. \tag{2}$$

We define $\tilde{k} = \sqrt{2m\tilde{\omega}/\hbar}$ with a branch cut slightly below the real axis. For $\tilde{k}_0 = \sqrt{2m\tilde{\omega}_0/\hbar}$ its real part is negative or zero and the imaginary part is zero or positive.

2.1. Dimensionless Schrödinger equation

We introduce dimensionless quantities in terms of a characteristic length L

$$x = \tilde{x}/L,$$
$$t = \tilde{t}\frac{\hbar}{2mL^2},$$
$$\psi(x, t) = \sqrt{L}\tilde{\psi}(\tilde{x}, \tilde{t}), \tag{3}$$

so that the Schrödinger equation takes the form