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# Reprint of : Effect of incoherent scattering on three-terminal quantum Hall thermoelectrics

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## HIGHLIGHTS

- Chiral thermoelectrics is affected by decoherence.
- Chirality ensures separated thermalisation of carriers.
- Energy harvesting and heat diode operations are robust in the presence of inelastic scattering.

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## ABSTRACT

A three-terminal conductor presents peculiar thermoelectric and thermal properties in the quantum Hall regime: it can behave as a symmetric rectifier and as an ideal thermal diode. These properties rely on the coherent propagation along chiral edge channels. We investigate the effect of breaking the coherent propagation by the introduction of a probe terminal. It is shown that chiral effects not only survive the presence of incoherence but they can even improve the thermoelectric performance in the totally incoherent regime.



## 1. Introduction

The last decades of the 20th century saw the development of the field of quantum transport in mesoscopic conductors. The quantum nature of electrical carriers shows up in systems of reduced dimensionality, where their phase coherence is maintained when being transported through the sample. The quantisation of conductance in quantum point contacts [1,2], the existence of persistent currents in normal metal rings [3–5], or the possibility to design electronic interferometers [6] are good examples. The scattering theory of mesoscopic conductors, developed after the ideas of R. Landauer, has been successfully used in many of these problems, where electron–electron interactions do not play a role. The formal elaboration of the theory was established by M. Büttiker by emphasizing the role of multi-terminal measurements [7], the magnetic field symmetries [8], and the importance of decoherence [9] and fluctuations [10]. For a recent review, see Ref. [11].

Another important achievement was the formulation of transport along quantum Hall edge-channels [12] which is not affected by back-scattering [13]. The quantum Hall effect manifests in four-terminal measurements in the presence of strong magnetic fields: together with the longitudinal injection of a current, a transverse resistance is measured that shows plateaus at inverse integer multiples of $h/e^2$ [14].

The effect of interactions can be modelled within the scattering matrix framework by introducing phenomenological probes [9]. They consist of one or more terminals whose coupling to the system mimics the desired effect. Voltage [9], dephasing [15,16] or thermometer probes [17–19] can be defined by considering the appropriate boundary conditions on (energy-resolved) charge and heat currents. For example, a voltage probe that injects no net charge current into the system introduces the effect of decoherence by inelastic scattering [9]. Electrons are absorbed by the probe and re-injected in the system at a randomised energy. The

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coupling to the probe defines a crossover between the purely coherent transport and the regime where the electron has lost its phase coherence when propagating between the conductor terminals. It may also lead to enhanced correlations [20,21]. Inelastic scattering is present, e.g., in a conductor coupled to a fluctuating environment.

The coupling to external fluctuations also generates correlations in the conductor [22,23]. They can be a source of transport, even if the conductor itself is in equilibrium. In order to rectify fluctuations from a non-equilibrated environment, the conductor must break electron–hole and left–right symmetries. Such conditions are generally present in mesoscopic circuits and nanojunctions. That is the origin of the mesoscopic Coulomb drag effect [24–26]: a current injected in a two terminal conductor generates a current in a second conductor to which it is capacitively coupled [27,28].

A related effect is found in three-terminal conductors if the non-equilibrium situation is induced by a temperature gradient [29]. It thus gives rise to a transverse thermoelectric effect. A current is generated between two terminals by the conversion of heat absorbed from the hot third terminal. The hot environment can be fermionic [29–31] or bosonic [32–35], provided that it does not inject charge into the system, cf. Ref. [36] for a recent review and Refs. [37–39] for recent experimental realisations. This effect can be described by a probe terminal which injects heat in the conductor by being maintained at a higher temperature [40–42]. The presence of the third probe can also be beneficial for the thermoelectric performance of the conductor [43].

The application of a magnetic field introduces a number of new phenomena [44–49]. In a recent work, we showed that the quantum Hall effect shows up in the thermoelectric response of a three-terminal configuration [45]. The appearance of chiral propagation along edge channels under strong magnetic fields has important consequences in the transverse thermoelectric response of the system [50,51]. In particular, a finite charge current is predicted in a left–right symmetric conductor as the one represented in Fig. 1. Furthermore, the system behaves as an ideal thermal diode [47]. The contributions responsible for these two effects remarkably depend on the coherent propagation between the two conducting terminals.

In this paper, we address the question of how much these effects are affected by decoherence. We do so by introducing a probe terminal that interrupts the propagation between the two terminals, cf. Fig. 1. Naïvely, one expects that the transition to a strongly coupled probe will bring the system to the sequential regime where chiral effects are suppressed. On the other hand, the presence of the probe emphasises the importance of having a non-equilibrium situation in the middle of the conductor. In our case, it is defined by the left and right moving carriers being thermalised by probes at different temperatures, $T_3$ and $T_p$. Hence, we combine the two possible uses of a voltage probe: one of them serves as a model for a non-equilibrium environment able to generate current while the other one acts as a source of decoherence.

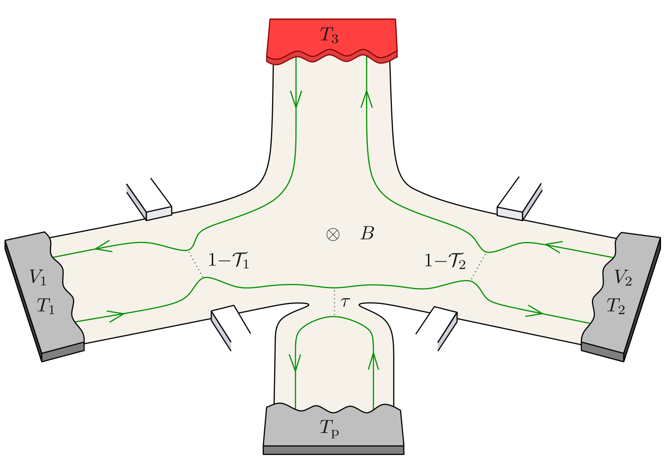


**Fig. 1.** Three-terminal quantum Hall thermoelectric device coupled to a voltage probe. Terminals 1 and 2 hold a charge current in the presence of a voltage bias $V = V_1 - V_2$, or a temperature gradient. The latter can either be applied longitudinally (at terminals 1 or 2) or transversally (at terminals 3 or $p$). Terminals 3 and $p$ are considered as probes whose voltage adjusts such that they do not inject charge into the system. The thermoelectric response relies on the energy dependence of the scattering at the constrictions, in our case quantum point contacts in terminals 1 and 2. The coupling $\tau$ to the probe affects the chiral contributions to the heat conduction.

## 2. Scattering theory

Electronic transport along non-interacting edge channels is well described by the Landauer–Büttiker formalism [13]. We will restrict ourselves to the case with a single edge channel. In this formalism, linear-response charge and heat currents $\mathbf{I}_i = (I_i^e, I_i^h)$ can be expressed in a compact form [7,52,53]:

$$\mathbf{I}_i = \frac{1}{h}\sum_j \int dE\,[\delta_{ij} - \mathcal{T}_{i\leftarrow j}(E)]\,\xi(E)\begin{pmatrix} e & eE \\ E & E^2 \end{pmatrix}\mathbf{F}_j, \tag{1}$$

in terms of the transmission probabilities $\mathcal{T}_{i\leftarrow j}(E)$ for electrons injected in terminal $j$ to be absorbed by terminal $i$, and the electric and thermal affinities $\mathbf{F}_j = (F_j^V, F_j^T)$, with $F_i^V = eV_i/(k_B T)$ and $F_i^T = k_B \Delta T_i/(k_B T)^2$. Here $V_i$ and $\Delta T_i$ are the voltage and the temperature bias applied to terminal $i = 1, 2, 3, p$, respectively, and $k_B T$ is the system temperature. We have introduced the derivative of the Fermi function $\xi(E) = -(k_B T/2)\,df/dE$. The equilibrium Fermi energy $E_F$ is considered in the following as the zero of energy.

Terminals 1 and 2 define the electric conductor which supports a charge current $I^e = I_1^e = -I_2^e$. Terminal 3 models a heat source. Terminal $p$ introduces inelastic scattering. Thus the latter two terminals inject heat but no charge (on average) into the conductor, i.e. $I_3^e = I_p^e = 0$ [9]. The voltage of terminals 3 and $p$ is left to accommodate to the configuration at which the probe boundary conditions are satisfied. All other voltages and temperatures are fixed. Charge and heat currents

$$I^e = GV + \sum_j L_{1j}\Delta T_j \tag{2}$$

$$I_i^h = M_{i1}V + \sum_j K_{ij}\Delta T_j \tag{3}$$

flow in response to a voltage bias $V = V_1 - V_2$ or to a thermal gradient applied to each other terminal. $G$ and $K_{ij}$ are the electrical and thermal conductances, respectively. The thermoelectric response is given by the Seebeck and Peltier coefficients, here proportional to $L_{1j}$ and $M_{i1}$, respectively. In the presence of a magnetic field, the linear response coefficients are known to be linked by the Onsager reciprocity relations [8,53–55]:

$$L_{1j}(B) = M_{j1}(-B)/T, \tag{4}$$

and by energy conservation, $\sum_j I_j^h = 0$.

We will focus here on the transverse Seebeck coefficient, $L_{13}$, and the longitudinal off-diagonal thermal conductances, $K_{12}$ and $K_{21}$. The former term gives rise to an electric current generated between terminals 1 and 2 by conversion of the heat injected from terminal 3 being at a higher temperature. This is the process of relevance for energy harvesting [36]. The latter coefficients give