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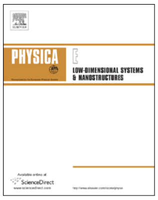

Nonlocal and surface effects on the buckling behavior of functionally graded nanoplates: An isogeometric analysis

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HIGHLIGHTS

- Isogeometric buckling analysis of FGM nanoplates.
- Considering different geometries including circular, elliptical and skew nanoplates.
- Considering size influences based on the Eringen and Gurtin–Murdoch theories.
- Developing a novel vector-matrix form of formulation with the capability of being easily used in the finite element method or isogeometric analysis.
- Investigating nonlocal and surface effects on the buckling responses of nanoplates.

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ABSTRACT

The size-dependent static buckling responses of circular, elliptical and skew nanoplates made of functionally graded materials (FGMs) are investigated in this article based on an isogeometric model. The Eringen nonlocal continuum theory is implemented to capture nonlocal effects. According to the Gurtin–Murdoch surface elasticity theory, surface energy influences are also taken into account by the consideration of two thin surface layers at the top and bottom of nanoplate. The material properties vary in the thickness direction and are evaluated using the Mori–Tanaka homogenization scheme. The governing equations of buckled nanoplate are achieved by the minimum total potential energy principle. To perform the isogeometric analysis as a solution methodology, a novel matrix-vector form of formulation is presented. Numerical examples are given to study the effects of surface stress as well as other important parameters on the critical buckling loads of functionally graded nanoplates. It is found that the buckling configuration of nanoplates at small scales is significantly affected by the surface free energy.

1. Introduction

Buckling and vibration behaviors of structural elements are extensively studied by researchers. For example, Wu et al. [1] investigated the post-buckling configuration of functionally graded plates in thermal environment. They used the simple power law distribution scheme for variation of material properties, and implemented Chebyshev polynomials to obtain the critical buckling loads and temperatures of plates with different boundary conditions. Huang and Han [2] presented the nonlinear pre- and post-buckling analysis of axially compressed functionally graded cylindrical shells based on the Ritz energy method and large deformation theory. Also, the free vibration behavior of functionally graded fluid-conveying orthotropic piezoelectric hollow cylinder is studied by Chen et al. [3]. A laminate model is employed to model the functionally graded shell, and problem formulations are presented based on the state equations. The buckling problem in the presence of an obstacle is also attracted the attention of some researchers. For instance, Bersani et al. [4] investigated the buckling of an axially symmetric elastic hemispherical shell, uniformly compressed, subject to a constraint to the radial shifting of the equatorial circumference.

In recent years, the mechanical analyses of micro- and nanostructures made of functionally graded materials (FGMs) have become attractive research topics (e.g. [5–7]). The nonlocal continuum theory [8–10] as a higher-order elasticity theory is capable of capturing size effects on the behavior of such structures. Unlike the classical (local) elasticity theories, in the nonlocal elasticity theory, the stress field at one point of a body depends on the strain field at all points. From published literature about nonlocal small scale plates, one can mention the work of Liu et al. [11] in which the electro-thermo-mechanical dynamic response of piezoelectric

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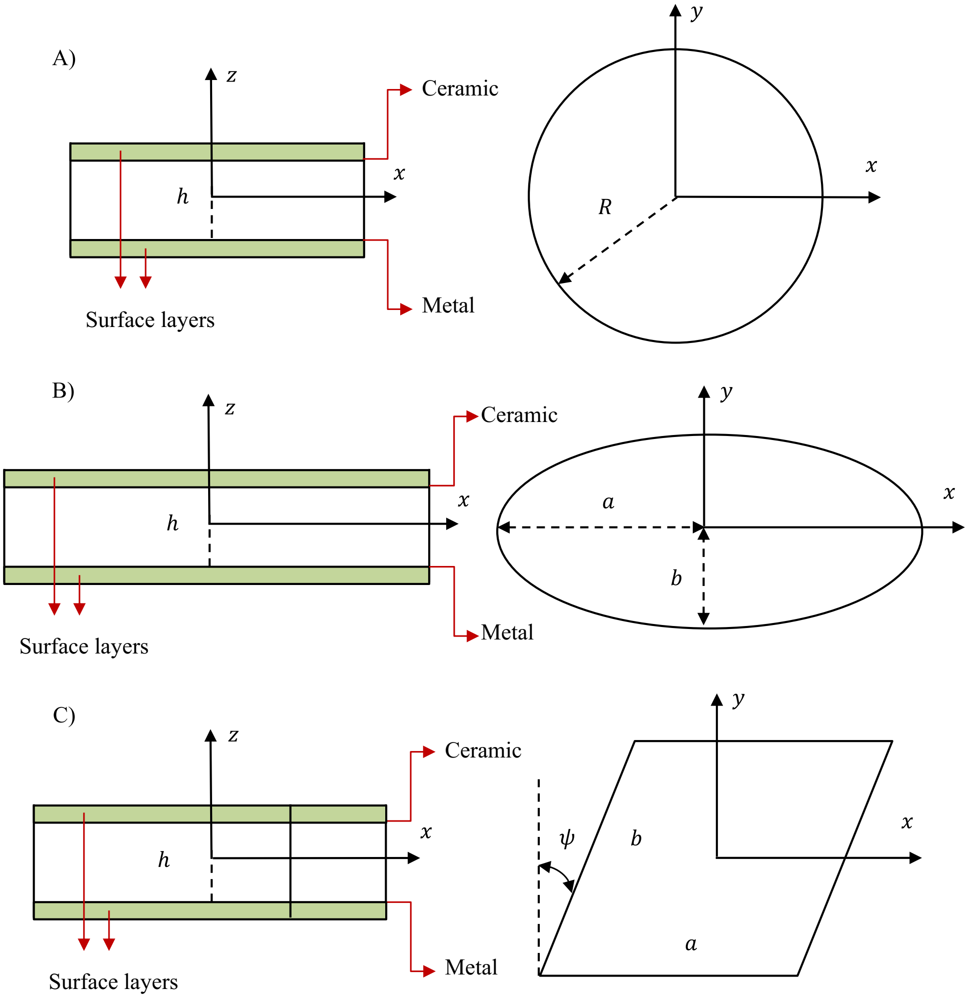

Fig. 1. Geometry of functionally graded (a) circular, (b) elliptical and (c) skew nanoplates with upper and lower surface layers.

Table 1.
Comparison of dimensionless critical bi-axial buckling load ($P_{cr}R^2/D$) of isotropic circular nanoplate with Refs. [53,14], $R = 10$nm,$h = 0.34$nm.

BC	$\bar{\mu}$	Present	Ref. [53] RPT	Ref. [53] Quasi 3D	Ref. [14]
Simply	0	4.1973	4.1920	4.2563	4.1976
	1	4.0314	4.0226	4.0798	4.0285
	2	3.8723	3.8664	3.9176	3.8725
	3	3.7230	3.7220	3.7680	3.7281
	4	3.5837	3.5880	3.6294	3.5941
Clamped	0	14.6727	14.6135	15.1679	14.6819
	1	12.7930	12.7424	13.2132	12.8023
	2	11.3404	11.2960	11.7021	11.3493
	3	10.1840	10.1446	10.4988	10.1925
	4	9.2416	9.2061	9.5175	9.2497

simply-supported nanoplates was analyzed based on the nonlocal elasticity theory. Narendar and Gopalakrishnan [12] studied the wave propagation of nanoplates based on the nonlocal continuum. They examined the influences of low and high temperature on the ultrasonic wave dispersion. Ansari et al. [13] analysed the free vibration characteristics of single-layered graphene sheets (SLGSs) by incorporating the nonlocal effects into the Mindlin plate theory. The generalized differential quadrature (GDQ) method and the molecular dynamic (MD) simulation are used to achieve natural frequencies corresponding to different boundary conditions and find the appropriate values of nonlocal parameter relevant to each boundary condition. The buckling behavior of local circular SLGSs subjected to uniform radial compressive loads was presented by Farajpour et al. [14]. Rouhi and Ansari [15] developed a nonlocal Flügge shell model for the buckling analysis of double-walled carbon nanotubes. They used the Rayleigh–Ritz method in their analysis. Also, Ansari et al. [16,17] investigated the axial buckling of carbon nanotubes and boron-nitride nanotubes by combining the nonlocal theory and molecular mechanics.

It has been reported that the material properties of the surface part of a solid differ from those of its bulk section [18,19]. So, it seems that the surfaces should be modelled separately and their influences should be considered in the analysis. These influences are more prominent at micro- and nano- scales or in cases with high surface-to-volume ratios [20–23]. Dingreville et al. [23] developed a continuum model with taking the free surface energy into account and studied the effects of surface layers on the elastic behavior of nanoparticles, nanofilms and nanowires. Wang and Feng [24] illustrated the effects of considering surface elasticity and residual surface tension on frequency responses of simply supported microbeams. Altenbach and Eremeyev [25] presented a