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Surface effect on the nonlinear forced vibration of cantilevered nanobeams

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HIGHLIGHTS

- A nonlinear model is proposed for cantilevered nanobeams with consideration of surface effect.
- Positive residual surface stress would decrease the natural frequencies.
- The nonlinear nanobeam system exhibits both hardening and softening behaviors.

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ABSTRACT

The nonlinear forced vibration behavior of a cantilevered nanobeam is investigated in this paper, essentially considering the effect due to the surface elastic layer. The governing equation of motion for the nano-cantilever is derived, with consideration of the geometrical nonlinearity and the effects of additional flexural rigidity and residual stress of the surface layer. Then, the nonlinear partial differential equation (PDE) is discretized into a set of nonlinear ordinary differential equations (ODEs) by means of the Galerkin's technique. It is observed that surface effects on the natural frequency of the nanobeam is of significance, especially for the case when the aspect ratio of the nanobeam is large. The nonlinear resonant dynamics of the nanobeam system is evaluated by varying the excitation frequency around the fundamental resonance, showing that the nanobeam would display hardening-type behavior and hence the frequency-response curves bend to the right in the presence of positive residual surface stress. However, with the negative residual surface stress, this hardening-type behavior can be shifted to a softening-type one which becomes even more evident with increase of the aspect ratio parameter. It is also demonstrated that the combined effects of the residual stress and aspect ratio on the maximum amplitude of the nanobeam may be pronounced.

1. Introduction

In the past decade, it has been reported that nano-scale structures with high surface-to-volume ratio exhibit superior mechanical, electrical and thermal performances compared to their micro- and/or macro-scale counterparts [1–3]. In micro/nano electromechanical systems (MEMS/NEMS), nanobeams have become one of the most important core components, widely used in many areas, including communications, information technology, machinery, biotechnology and etc. [4,5]. Among various applications, an important scientific problem that has attracted considerable attention in the literature is the static and dynamic behaviors of nanobeams due to the surface effects and/or small scale effects.

According to the classical/conventional continuum mechanics, the effect of surface layer may be ignored as they are small enough compared to the bulk material. For nano-scale materials/structures, however, the effects of surface layer may be remarkable since the surface-to-volume ratio becomes high [1]. The surface-to-volume ratio, indeed, has been demonstrated to play a critical role in nano-sized problems via atomistic simulations and experimental evaluations, showing the importance of accounting for surface effects. A number of excellent collections on the mechanics of nano-structured materials with consideration of surface effect

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can be found in the comprehensive review by Wang et al. [6].

To describe the surface effects on the mechanical responses of nano-sized materials/structures, the so-called surface elasticity theory has been developed by modeling the surface as a two dimensional membrane adhering to the underlying bulk material without slipping [7,8]. It is reported that the predictions based on the theory of surface elasticity established by Gurtin et al. [8] fit well with atomistic simulations and experimental measurements [9–11]. Indeed, there were a number of studies utilizing the surface elasticity theory to analyze the static and dynamic behavior of nanobeams based on different beam assumptions. The buckling instability, wave propagation, free vibration, and nonlinear vibrations of nanobeams including surface effects are of interest in the past several years.

For slender nanobeams/nanowires (the length is much larger than the thickness), the Euler–Bernoulli beam assumption holds. In 2009, Wang and Feng [12] investigated the surface effects on the axial buckling of slender nanowires. They obtained an analytical expression of the critical force at which the nanowire occurs axial buckling to account for both the effects of surface elasticity and residual surface tension. Based on the Laplace–Young equation, Wang and Feng [13] and He and Lilley [14] further addressed both the impacts of residual surface stress and surface elasticity on the free vibration of slender nanobeams, which has been verified by experiments [9–11]. By using the surface Cauchy–Born model, Park [15] studied the size-dependent effect of the residual surface stress on the resonant frequencies of silicon nanowires under finite deformation. What's more, Wang [16] utilized the surface elasticity theory to study the instability and vibration characteristics of slender nanobeams containing internal fluid flow, showing the significance of surface effect on the critical flow velocity and natural frequency If, however, the length of nanobeams is insufficiently long to be simplified as an Euler–Bernoulli beam, the Timoshenko beam theory has been employed to analyze the axial buckling [17]and free vibration [18] of nanobeams.

It should be noted the previously mentioned studies treating nanobeams with surface effects are mostly based on linear theories. In recent years, nonlinear theories have been utilized to develop nonlinear models for predicting statics and dynamics of nanobeams like the postbuckling behavior, nonlinear free vibration, nonlinear forced vibration, and pull-in instability due to electrical loading with consideration of surface effects. Ansari et al. [19,20] predicted the postbuckling characteristics of supported nanobeams in the presence of surface stress within the framework of Euler–Bernoulli and Thimoshenko beam theories, respectively. They also investigated the surface effects on the nonlinear free vibration [21] and nonlinear forced vibration [22] of supported nanobeams using surface elasticity theory. It was revealed that by incorporating the surface stress effect, the maximum amplitude occurs at lower excitation frequencies and the wide of region of the response tends to decrease [22]. Hosseini-Hashemi et al. [23] considered the surface effects on the nonlinear free vibration analysis of simply-supported functionally graded (FG) nanobeams using nonlocal elasticity theory. The multiple scale method was employed as an analytical solution for the nonlinear governing equation to obtain the nonlinear natural frequencies of FG nanobeams. Sahmani et al. [24] further studied the free vibration characteristics of postbuckled third-order shear deformable FG nanobeams including surface effects. It was revealed that the surface effect plays an important role on the vibration characteristics of the buckled FG nanobeams with low thicknesses. Fu and Zhang [25] studied the static pull-in phenomena in electrically actuated nanobeams with fixed-fixed boundary conditions using the surface elasticity theory. Numerical results indicated that the pull-in phenomena are size-dependent. Recently, Wang and Wang [26] constructed a theoretical model for nano-cantilever switches with consideration of surface effect and nonlinear curvature. They analyzed the static pull-in instability mechanism due to electric loadings and found that the surface effect can be significant. More recently, Dai and Wang [27] further investigated the dynamic pull-in instability of cantilevered nanobeam using a full nonlinear model including inertia nonlinearity. It was shown that the surface effect on the dynamic pull-in instability is pronounced when the surface-to-volume ratio becomes high.

From the literature survey, it ought to be noted that most of the available studies on the nonlinear dynamical behaviors of nanobeams given surface effects were focused on the system with positively supported ends. Very few papers concerned on the nonlinear dynamical behaviors of cantilevered nanobeams with surface elastic layer [26,27]. For a nanobeam with positively supported ends, one may clearly consider the nanobeam to be extensible, that is, the length of its centerline is varied during oscillation, resulting in its lateral deflection. However, when the nanobeam has clamped-free boundary conditions, inextensibility of the nanobeam is easily understood with the length of its centerline remaining constant during oscillation. Therefore, this significant difference of characteristic between supported and clamped-free nanobeams leads to the corresponding nonlinear governing equations fundamentally different. Then, the nonlinear vibrations of supported and cantilevered nanobeams should be separately treated. According to the authors' literature review, the effect of surface layer on the nonlinear forced vibration of nanobeams with clamped-free boundary conditions has not been identified yet.

The main purpose of the current work is to develop a nonlinear model for investigating the surface effects on the nonlinear forced vibrations of nanoscale Euler–Bernoulli beams based on the surface elasticity theory. Hamilton's principle is utilized to derive the size-dependent nonlinear governing equation of motion which is solved numerically on the basis of Galerkin's technique and Runge–Kutta method. Subsequently, the surface effects on the natural frequencies and nonlinear dynamic responses are examined for cantilevered nanobeams subjected to external excitations.

2. Governing equation

The system under consideration consists of a slender nanobeam with rectangular cross section. The nanobeam is cantilevered with length L, as shown in Fig. 1(a). In addition, the nanobeam is considered to have an elastic surface perfectly bonded to

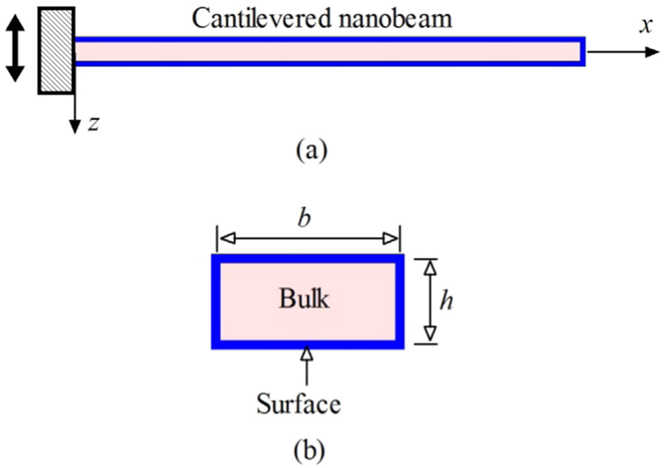

Fig. 1. Schematic of the nanobeam with surface elastic layer.