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Modeling of non-*Darcy* flow through anisotropic porous media: Role of pore space profiles

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HIGHLIGHTS

- The effect of curvature of throats on intrinsic properties of the porous media is investigated.
- Conical, parabolic, hyperbolic and sinusoidal throat profiles are used and compared.
- Analytical solution of 1D *Navier–Stokes* equations for converging/diverging pores is applied.
- The non-*Darcy* coefficient mostly depends on throat curvatures while permeability is not.
- Non-*Darcy* behavior is mostly affected by throat radius than pore radius and throat length.
- Pore morphology and anisotropy of the network are taken into account.

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ABSTRACT

Excess pressure drop induced by inertial effects limits the applicability of *Darcy*'s law for modeling of fluid flow through porous media at high velocities. It is expected such additional pressure drop is influenced by pore/morphology of porous media. This work concerns with fundamental understanding of how throat curvature affects intrinsic properties of porous media at non-*Darcy* flow conditions using network modeling. Conical, parabolic, hyperbolic, and sinusoidal capillary ducts with three types of imposed anisotropy are used to construct the network in a more realistic manner. Solutions of one dimensional *Navier–Stokes* equation for incompressible fluid flow through converging/diverging pore geometries have been utilized to acquire the pressure drop versus volumetric flow rate to investigate the role of various pore space profiles on the properties of porous media, which make the results of this work different from previous studies in the literature. Macroscopic inherent parameters of porous media such as tortuosity, porosity, permeability as well as non-*Darcy* coefficient are evaluated as outputs of the model. It has been revealed that the non-*Darcy* coefficient mainly depends on throat curvatures while permeability is not. While average throat radius is constant, both permeability and non-*Darcy* coefficient are increasing with average body radius. Among induced anisotropies, alteration of throat radius is the most effective parameter on non-*Darcy* coefficient. Regarding the throat morphology, some new general correlations for predicting the non-*Darcy* coefficient as a function of porosity, tortuosity, permeability, and the ratio of diverging/converging tubes in the network have been proposed. Results of this study could help better understanding of how the morphology of pores/throats affects the non-*Darcy* coefficient.

1. Introduction

Pressure drop along porous media depends on the local velocity of fluid passing through it; this means that the amount of pressure loss is a function of volumetric flow rate (Ergun, 1952). Skjetne (1995) categorized different flow regimes for simple geometrical systems as: *Darcy*, weak inertia, strong inertia, and turbulence. With increasing the velocity of fluid moving throw porous medium, the fluid regime approaches turbulence. In petroleum reservoirs, for regions far from the well, turbulent flow occurrence is doubtful. However, near the fractures (Balhoff and Wheeler, 2009) or near the wellbore especially in gas reservoirs (Kalaydlian et al., 1996), observation of strong inertia flow behavior is not an unexpected phenomenon.

At low flow velocities, there is a linear proportionality between pressure drop and flow rate which is known as *Darcy*'s law (Wang

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et al., 1999). As the flow rate increases, the inertial effect in the *Navier–Stokes* equation becomes significant; accordingly, the pressure drop is no longer a linear function of flow rate which, not necessarily, means that the flow is turbulence (Thauvin and Mohanty, 1998). Since there is just non linearity in laminar flow, it would be better to consider this flow regime as a non-*Darcy* flow and not a turbulence one (Belhaj et al., 2003). It is widely accepted that *Forchheimer*'s equation (Forchheimer, 1901) can fulfill this divergence from linearity due to inertial effects which is expressed as follows:

$$-\frac{\Delta P}{L} = \frac{\mu}{k}v + \beta\rho v^2. \tag{1}$$

where $\Delta P/\Delta L$ is the pressure gradient, μ is viscosity, K is the permeability of the rock, ρ is the density of fluid, v is the velocity, and β is non-*Darcy* coefficient, inertial coefficient or inertial resistance. At low flow rates, the quadratic term in the right hand side of Eq. (1) is negligible and accordingly, Eq. (1) can be simplified to *Darcy* equation. Some researchers have mentioned that the *Forchheimer*'s equation can satisfactorily fit the experimental data of pressure drop at high fluid velocities (Ahmed, 1967; Fancher and Lewis, 1933; Kim, 1985; Sunada, 1965). Conversely, some other experiments have shown that this equation is not adequate for fitting data in some ranges of high flow rates (Barree and Conway, 2004; Forchheimer, 1930). Also, Forchheimer (1930) suggested that the terms with higher degree in Eq. (1) might better fit such data.

Many criteria have been proposed as the upper limit of *Darcy*'s flow. These criteria are usually expressed by *Reynolds* number defined in different ways and ranges. According to the geometry of pores, the most admissible upper limit of laminar flow is *Reynolds* numbers between 1 and 10.

For instance, Tek (1957) clarified that the upper limit is 1 or deVries (1979) limited the laminar flow to *Reynolds* number of 5. Their *Reynolds* number is on the basis of the average velocity of fluid and grain size. Hassanizadeh and Gray (1987) introduced a range for critical *Reynolds* number between 1 and 15, and recommended that 10 might be the best criterion. Additionally, Thauvin and Mohanty (1998) applied pore network modeling with new definition of *Reynolds* number as:

$$\mathrm{Re} = \frac{\rho r v}{\mu} \tag{2}$$

where r is the throat radius, and v is the throat velocity. They pointed out that the critical *Reynolds* number is 0.11. Also, Stark (1972), applied numerical experiments including the solution of *Navier–Stokes* equation for simple porous media, and found the critical *Reynolds* number between values of 5 and 13.

Pursuant to Eq. (1), plotting the graph of $\Delta P/Lv\mu$ versus $\rho v/\mu$, represents a straight line with the slope of $1/K$ and the intercept of β. Nevertheless, most of the time, it is a time consuming task to obtain the non-*Darcy* coefficient experimentally which is not a routine core analysis activity. Therefore, correlating the non-*Darcy* coefficient to the other properties of the rock is helpful. Owing to that the non-*Darcy* coefficient is an intrinsic property of the porous media, it is wildly held that β is a function of other innate parameters of the rock such as permeability and porosity (Coles and Hartman, 1998; Geertsma, 1974; Jones, 1987). It is accepted that the best empirical correlation which relates the permeability to non-*Darcy* coefficient can be yield as:

$$\beta = c_1 K^{c_2}. \tag{3}$$

According to the properties of porous media, e.g. pore size distribution and pore connectivity, the values of c_1 and c_2 can be determined. Some other correlations relate the non-*Darcy* coefficient to the permeability and porosity of media, simultaneously (Ergun, 1952; Janicek and Katz, 1955). Typical instance of such correlation is expressed by Geertsma (1974) for consolidated rocks in presence of immobile saturation of fluid as:

$$\beta = \frac{1}{K^{0.5}\phi^{5.5}}. \tag{4}$$

The major shortcomings of these correlations include that they are mostly obtained by bead or sand packs and do not contain the tortuosity as an affecting parameter. Since experimental correlations could not be a good reflection of pore-scale properties, they are just a relation between pressure drop along porous media and volumetric flow rate.

Along with the experimental investigations and due to their deficiencies, many researchers have tried to study the non-Darcy flow using the network modeling. Network modeling refers to representation of porous media as conceptual interconnected framework of pore and pore throats. Meanwhile, approximations to void space morphology and fluid mechanics dominant relations let modeling be accomplished in pore-scales which is not possible by empiricism.

Porous media has the high degree of complexity in terms of geometry (Caruso et al., 1985; Wang et al., 1999); accordingly, an uncomplicated representative model which approximately contains the properties of the natural sample in mathematically utilizable form would help studying the characteristics of the porous media. First network models were consisted of a bundle of tubes without any connection. These models were somewhat successful in predicting the inherent properties of porous media; however, they could not be effective in studying a lot of phenomenon such as mixing effect (Martines et al., 2007). Henceforth, some modifications with the purpose of improving this model were presented by researchers (Lahbabi and Chang, 1986; Pendse et al., 1983). Later researchers tried to modify this model by adding restriction inside the tubes or using the tubes with variable wall profile. For the first time Fatt (1956) proposed a network made up of a connected geometry in order to investigate two-phase flow through porous medium.

Literature of this field is full of cases which have used the interconnected system as a more practical depiction of natural samples (see for instance, Koplik, 1982, or Ioannidis and Chatzis, 1993) which serves many purposes comprising studying permeability (Bryant et al., 1993) or multi-phase flow (Leonormand et al., 1988). Lopez et al. (2003) modeled passing of non-Newtonian fluid through porous media. Blunt et al. (2002) used pore network to simulate the multi-phase flow in a representative geological porous media. Similarly, Valvatne and Blunt (2004) forecasted the properties of two-phase fluid flow considering the wettability of porous medium with models based on the realistic geological samples. In contrast with other researchers who mainly related the pressure drop to throat roughness, some others did not neglect the effect of pore restriction to better predicting of relative permeability during two phase flow (Raoof and Hassanizadeh, 2012). In aforementioned studies, attention is mostly given to understanding the behavior of multi-phase flow in porous media at creeping flow condition, but they did not discussed the role of pore morphological effects on the porous media's characteristics especially at high fluid velocities.

Nevertheless, there are some other researchers interested in examining of non-*Darcy* flow through porous media using pore network modeling. A two-dimensional random network was applied by Lao et al. (2004) to evaluate non-*Darcy* coefficient. Thauvin and Mohanty (1998) and Wang et al. (1999) have used a model consisted of pores and cylindrical throats with diverse lengths and radii to anticipate flow behavior at high velocities, and