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Nonlinear vibration of carbon nanotube embedded in viscous elastic matrix under parametric excitation by nonlocal continuum theory

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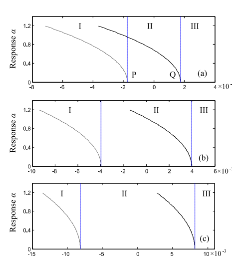
HIGHLIGHTS

- The amplitude–frequency response is presented by the multiple scale method.
- The gap between negative and positive bifurcation points can be enhanced by parametric load.
- The nonlocal continuum theory can present a more proper model.

GRAPHICAL ABSTRACT

It can be observed that the gap between two bifurcation points becomes wider with the axial load increasing, which means the parametric excitation can enhance the stable region.

Relation between the detuning parameter and response of principle parametric resonance for nanotube with the effects of parametric excitation. (a) $F=0.0005\ EA_c$, (b) $F=0.001\ EA_c$ and (c) $F=0.002\ EA_c$.



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ABSTRACT

In the present work, the nonlinear vibration of a carbon nanotube which is subjected to the external parametric excitation is studied. By the nonlocal continuum theory and nonlinear von Kármán beam theory, the governing equation of the carbon nanotube is derived with the consideration of the large deformation. The principle parametric resonance of the nanotube is discussed and the approximation explicit solution is presented by the multiple scale method. Numerical calculations are performed. It can be observed that when the mode number is 1, the stable region can be significantly changed by the parametric excitation, length-to-diameter ratio and matrix stiffness. This phenomenon becomes different to appear if the mode number increases. Moreover, the small scale effects have great influences on the positive bifurcation point for the short carbon nanotube, and the nonlocal continuum theory can present the proper model.

1. Introduction

With the superior characteristics for the large Young's modulus, yield strength, flexibility, and conductivity properties, nanostructures are widely applied to nanoelectronics, nanodevices, nanosensors and nanocomposites [1–6]. Because the atomistic approach is much time-consuming and it is rather difficult to perform the experiment at the nanoscale, the classical mechanics method is widely accepted due to its computational efficiency and simplicity.

Different from the classical continuum model, the nonlocal elastic theory presented by Eringen [7,8] assumes that the stress at a point is a function of strains at all points in the system. Then the nonlocal continuum theory can effectively describe the small scale effects at the nanoscale. As a result, quite a lot of investigations are devoted to the mechanical properties of nanotubes by the nonlocal

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continuum theory, including buckling [9–13], wave propagation [14–21] and vibration [22–34] behaviors.

Although much work has been reported by the nonlocal continuum model, they are mainly focused on the mechanical properties of the carbon nanotube with linear problems. Besides several researches on the nanotube with nonlinear characteristics [35–41], a lot of nonlinear problems for carbon nanotubes under the external excitation need to be considered in the future work. On the other hand, it is known that during the nonlinear forced vibration, the structure under the parametric excitation can show the unstable region. However, such a typical phenomenon has not been reported on the carbon nanotube, which is mainly because of the quite emerging subject on the nanotube with nonlinear vibration.

This wok is focused on the properties of the stable region for the nanotube under the external parametric excitation. By the nonlocal continuum theory and the multiple scale method, the governing equation is derived and the principle parametric resonance is analyzed. This work is expected to be helpful for the design and analysis of the nanoscaled structures.

2. Equations and derivations

As shown in Fig. 1, the carbon nanotube embedded in the viscous elastic matrix is subjected to the parametric excitation with the harmonic frequency Ω. The elastic stiffness and the damping coefficient of the matrix are k_w and μ. The length of the carbon nanotube is L and the deformation is along the z-axis with the displacement denoted as w.

Based on the nonlocal continuum theory presented by Eringen [7,8], the constitutive relation with the form of the integral equation is

$$\tau_{kl}(\mathbf{x}) = \int_V \alpha(\mathbf{x}, \mathbf{x}')\sigma_{kl}(\mathbf{x}')dV(\mathbf{x}'), \tag{1}$$

where τ_{kl} is the nonlocal stress tensor, σ_{kl} the local stress tensor, $\alpha(\mathbf{x}, \mathbf{x}')$ the kernel function which describes the influence of the strain at various location $\mathbf{x'}$ on the stress at a given location $\mathbf{x}$ and V the entire body.

Due to the difficulty to deal with the integral form in Eq. (1), its differential expression is usually applied as [30]:

$$[1 - (e_0a)^2\nabla^2]\boldsymbol{\sigma} = \mathbf{C}_0 : \boldsymbol{\varepsilon}, \tag{2}$$

where $\mathbf{C}_0$ is the elastic stiffness matrix of the classical elasticity, $\boldsymbol{\varepsilon}$ the strain vector, e_0 the constant appropriate to each material and a the internal characteristic length (e.g. the length of C–C bond, the lattice spacing and granular distance, etc.). The value of e_0 is determined from experiments or by matching dispersion curves of plane waves with the atomic lattice dynamics. And e_0a means the scale coefficient which denotes the small scale effect on the mechanical characteristics of nanostructures.

For the flexural vibration of the carbon nanotube, the relation between the stress and strain for one-dimensional state can be expressed as the following form:

$$\sigma_x - (e_0a)^2\frac{\partial^2\sigma_x}{\partial x^2} = E\varepsilon_x, \tag{3}$$

where E is Young's modulus.

Based on the Euler–Bernoulli beam model, the axial force and the resultant bending moment can be expressed as

$$N = \int_A \sigma_x dA, \quad M = \int_A z\sigma_x d\,A, \tag{4}$$

where z is the transverse coordinate measured in the deflection direction and A the area of the cross section of the nanotube.

The displacements have the following forms:

$$u_x(x, z, t) = u(x, t) - z\frac{\partial w}{\partial x}, \quad u_y = 0 \quad u_z(x, z, t) = w(x, t), \tag{5}$$

where u and w are the axial and transverse displacements, respectively.

For the nonlinear vibration with a large amplitude, the nonzero von Kármán nonlinear strain should be considered and the relation between the strain and displacement is given by

$$\varepsilon_0 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \varepsilon_1 = -z\kappa, \tag{6}$$

where ε_0 is the nonlinear extensional strain, $\kappa = -\partial^2 w/\partial x^2$ the bending strain and ε_1 the strain induced by κ.

Then, the von Kármán nonlinear strain (i.e. ε_{non}) is

$$\varepsilon_{non} = \varepsilon_0 + \varepsilon_1 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2}, \tag{7}$$

The equation of motion can be given as [42,43]

$$\frac{\partial S}{\partial x} = \rho A\frac{\partial^2 w}{\partial t^2} + F\cos\Omega t\frac{\partial^2 w}{\partial x^2} + k_w w + \mu\frac{\partial w}{\partial t}, \tag{8}$$

where S is the shear force, ρ the mass density of the nanotube, $k_w w$ denotes the load on per unit axial length from the matrix which can be described as the Winkler model and k_w is the material constant which is determined by the elastic matrix.

From Eqs. (3)–(6), the axial load and the bending moment can be expressed as

$$N - (e_0a)^2\frac{\partial^2 N}{\partial x^2} = E\,A\varepsilon_0, \tag{9a}$$

$$M - (e_0a)^2\frac{\partial^2 M}{\partial x^2} = E\,I\kappa, \tag{9b}$$

where $I = \int_A z^2 d\,A$ is the moment of inertia.

Then the nonlinear vibration equation for the nanotube under the axial parametric load is

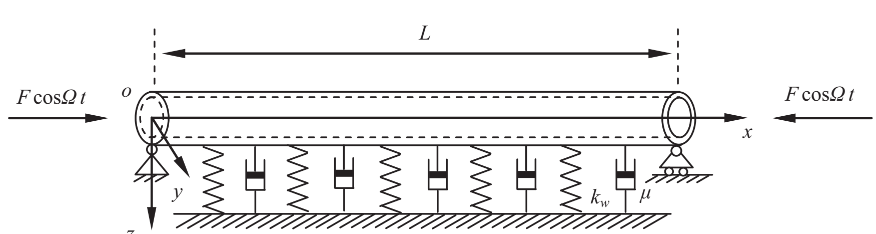

Fig.1. Nanotube embedded in viscous elastic matrix under the parametric excitation.