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# Effects of Pauli, Rashba and Dresselhaus spin–orbit interactions on electronic states in 2D circular hydrogenic anti-dot

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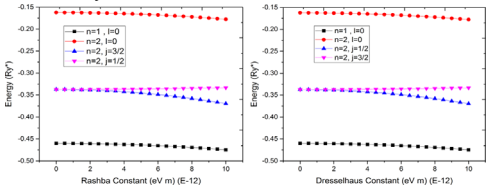
## HIGHLIGHTS

- Effects of spin–orbit couplings on electron's energy states, in *anti-dots*, are given.
- Pauli, Rashba and Dresselhuas interactions are explicitly taken into account.
- At the center of 2D circular anti-dot a hydrogenic donor is present.
- The significant degenerate states $j = 3/2$ and $j = 1/2$ are separated.
- Lift of degeneracy is in opposite sense for Rashba and Dresselhaus couplings.

## GRAPHICAL ABSTRACT

In the present article the effects of Pauli, Rashba, Dresselhaus spin–orbit interactions on the energy levels of a two-dimensional circular hydrogenic quantum *anti-dot* is considered. We show how these naturally arising interactions shift and remove (partially) the angular momenta (including spin) states. The corrections due to the three effcts are calculated and, through some illustrative figures, are physically interpreted. A representative figure in which the Rashba and Dresselhaus corrections are compared follows. From the figure it is clear that the degeneracy in the two states, $j = 3 = 2$, $m_j = \pm 3/2$ and $j = 1/2$, $m_j = \pm 1/2$ is removed, in the opposite sense, for the two interactions.

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## ABSTRACT

The present work is concerned with a report on the effects of Pauli, Rashba and Dresselhaus spin–orbit interactions (SOI) on the energy levels of a 2D circular hydrogenic quantum *anti-dot*(QAD). To pursue this aim, we first present a brief review on the analytical solutions to the Schrödinger equation of electronic states in a quantum anti-dot when a hydrogenic donor is placed at the center, revealing the degeneracies involved in the ground, first and second excited states. We then proceed by adding the aforementioned spin–orbit interactions to the Hamiltonian and treat them as perturbation, thereby, calculating the energy shifts to the first three states. As we show, the Rashba spin–orbit interaction gives rise to a shift in the energies of the ground and second excited states, while it partially lifts the degeneracy of the first excited state. Our calculations also indicate that the Dresselhaus effect, while keeping the degeneracy of the ground and second excited states intact, removes the degeneracy of the first excited state in the opposite sense. The Pauli spin–orbit interaction, on the other hand, is diagonal in the appropriate bases, and thus its effect is readily calculated. The results show that degeneracy of $\ell = 0$ (prevailing in the ground and second excited state) remains but the degeneracy of $\ell = 1$ (prevailing in the first excited state) is again partially lifted. Moreover, we present the energy corrections due to the three spin–orbit interactions as functions of anti-dot's radius, Rashba and Dresselhaus strengths discussing how they affect the corresponding states. The material presented in the article conceives the possibility of generating spin currents in the hydrogenic circular anti-dots.



## 1. Introduction

Manipulation of the electronic (hole) states, in particular the spin, without the application of external magnetic fields, is currently one of the fundamental challenges in the field of spintronics

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which takes advantage of spin states instead of the charge [1–4]. The desire for the development of spintronic devices stems from the well-established fact that such would decrease the power consumption, while drastically increase the efficiency and speed of information processing [5,6]. In fact, these significant properties of spintronic devices have paved the way for applications in the field of information processing [4,7], ranging from storing [6], reading [8], encoding and decoding [4,9], transmission [4,5,7], etc. [10] of data. To this end, the major task is the generation of spin-polarized currents from a nonpolarized one, in a controllable environment, along with appropriate means of manipulation [11,12]. A vivid candidate to attain these aims is semiconductor heterosturctures, mainly due to the fact that fabrication of low-dimensional semiconductor structures has advanced in the last decade [12]. A direct consequence of such advancement is the reduction of effective dimensions of the electronic surrounding from three dimensional bulk to quasi-two dimensional quantum wells [13,14], quasi-one dimensional quantum wires [13,15], and even quasi-zero dimensional quantum (anti)dots [13,14,16–18]. Amongst these quantum structures, anti-dots have been proposed for a wide range of application including, to name a few, the development of optoelectronic devices [17,19], spin injectors, spin filters [20], etc. [21,22]. The research on the behavior of electronic energy states has also been extended to investigation of dots and anti-dots prepared in graphene [23–25]. These extraordinary properties of such nanostructures, in particular the anti-dots, stem from the fact that quantum confinements lead to electronic characteristics completely different from those in bulk semiconductors [26,27]. It is thus the main aim of the present report to explore the electronic energy states when it is confined to move everywhere but inside a two dimensional dot, forming an anti-dot. In our consideration, moreover, we pay due attention to the electronic spin–orbit interactions, naturally occurring at the interface of two semiconductors where the anti-dot is formed. To be more specific, in what follows the Pauli, Rashba and Dresselhaus spin–orbit interactions are fully accounted for.

It is by now well established that in a heterojunction where the anti-dot is formed and the electrons reside, two fundamental spin–orbit interaction (SOI) naturally appear: the Rashba [28–32] and Dresselhaus [29,30,32,33] interactions. The former is known to arise when the structural space inversion symmetry in a crystal is broken (as in heterojunctions), while the latter is due to the bulk inversion asymmetry. It is also well established that the structural inversion asymmetry (consequently, the Rashba SOI) and the bulk inversion asymmetry (consequently, the Dresselhaus SOI) depend upon the band structure of the material, the electron density and the actual geometry of the sample [1]. It has been also demonstrated that the strength of Rashba SOI can be controlled with an electrostatic potential applied via a top gate terminal while that of the Dresselhaus SOI depends on the side-wise external gate potentials [1,30]. It is emphasized that for an anti-dot produced at the interface of GaAs/AlGaAs, both effects are present and may be tuned to be of equal strength in the range of $0–10\times10^{-12}$ eV m [34]. Equal Rashba and Dresselhaus spin–orbit interaction parameters are of great interest in construction of spin-field-effect transistors [1]. In the remaining of the present discussion it is assumed that the anti-dot is formed in the interface of GaAs/AlGaAs. The origin of naturally occurring Pauli spin–orbit interaction, as any text book on Quantum Mechanics does confirm [35], is due to the electronic motion in an electrostatic field. The main task in what follows, therefore, is to include all three types of spin–orbit interactions into the electronic Hamiltonian, subjected to the electrostatic field of a hydrogenic impurity (donor), and calculate the energy states.

Although there have recently been numerous reports on the behavior of charge carriers confined to quantum dots [36,37], even in the presence of impurities [38,39], relatively less attention has been devoted to that in quantum anti-dots [26,27,40,41]. However, due to recent advances in the techniques of semiconductor growth, anti-dots have been readily fabricated [42,43] and thus deserve a wider examination. This point of interest is much supported by the fact that quantum anti-dots have been proposed for practical applications in the development of, for instance, microwave resonators [21] and optoelectronic devices [19]. To this end, the presence of impurities is an unavoidable fact. If the impurity is of the IV group of elements, which is usually the case in the growth of GaAs/AlGaAs [44], the effect is modeled as a hydrogenic donor, giving rise to an attractive electrostatic Coulomb potential. In addition to the three SOI's, in the present work we also include the effects of such impurities. With these presumptions, we shall solve, perturbatively, the Schrödinger equation for an electron confined *outside* a circular disk (anti-dot) subject to an attractive Coulomb potential, Pauli SOI, Rashba SOI and Dresselhaus SOI. It is henceforth shown that under the action of any of the three SOIs, the two-fold degenerate $\ell = 0$ state remain degenerate, with just a shift in the energies (except the Pauli SOI, giving a zero shift), while the degeneracy of the $\ell = 1$ state is partially removed by either of them. The latter point, as we demonstrate, is accompanied by shifts in energies as well. Moreover, our calculations show that as the radius of anti-dot increases, while other parameters are held fixed, the eigenenergies also *increase*. This unexpected result is in sharp contrast with that in ordinary dots [39]. As expected, however, the growth in energies ceases after a critical radius which depends upon other parameters involved. Along these lines we also provide a thorough discussion of the effects of Rashba and Dresselhaus SOI strengths, which are externally controllable, on the energy shifts as well. The outcome of the present work, consequently, offers new means of controlling the electronic orbital angular momentum and spin states.

The present work is organized in the following manner. In the next section we present a discussion of the physical model, the governing Hamiltonian and the unperturbed states. In this section we also introduce the three naturally arising spin–orbit interactions into the unperturbed Hamiltonian. In Section 3 the energy eigenvalues, up to the second order of perturbation, are calculated and effects of the anti-dot's radius and the three spin–orbit interaction strengths are given. For more clarity, graphs of energy eigenvalues versus the aforementioned parameters are also given in this section. Interpretation of the results and concluding remarks are presented in the last section.

## 2. Physical model, the governing Hamiltonian and the unperturbed states

The bare Hamiltonian of an electron in a 2D circular quantum anti-dot of radius $R_\circ$, in the presence of hydrogenic donor impurity, situated at the center and in the effective mass approximation, is given by [17],

$$H_\circ = \frac{p^2}{2m^*} + V(r), \tag{1}$$

where $m^*$ is the electron effective mass and $V(r)$ includes the Coulomb and the confining potentials:

$$V(r) = \begin{cases} V_\circ - \dfrac{e^2}{\varepsilon r} & r < R_\circ \\ -\dfrac{e^2}{\varepsilon r} & r > R_\circ \end{cases}, \tag{2}$$

In Eq. (2) $\varepsilon$ is the permittivity of the medium, which is determined by all electromagnetic (screening) effects, and $V_\circ$ is the