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Nonlocal three-dimensional theory of elasticity with application to free vibration of functionally graded nanoplates on elastic foundations

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HIGHLIGHTS

- Analyzing vibrations of FG nanoplates with different edge conditions on elastic foundation.
- Development of a size-dependent plate model based on a 3D nonlocal theory of elasticity.
- Simultaneous discretization of governing equations in all three coordinate directions via GDQ.
- Exploring the influence of various model parameters on natural frequency of nanoplate.

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ABSTRACT

In the present work, a three-dimensional (3D) elastic plate model capturing the small scale effects is developed for the free vibration of functionally graded (FG) nanoplates resting on elastic foundations. The theoretical model is formulated employing the nonlocal differential constitutive relations of Eringen in conjunction with the 3D equations of motion of elasticity.The material properties are assumed to vary continuously along the thickness of the nanoplate in accordance with the power law formulation. Through extending the generalized differential quadrature (GDQ) method to the three-dimensional case, the governing equations are simultaneously discretized in every three coordinate directions and are then recast to the standard form of an eigen value problem. Solving the acquired problem, the natural frequencies of the nanoplates with different boundary conditions are calculated. The convergence behavior of the numerical results is checked out and comparison studies are conducted to make sure of the accuracy and reliability of the present model. Finally, the dependence of the vibration behavior of the nanoplate on edge conditions, elastic coefficients of the foundation, scale coefficient, mode number, material and geometric parameters are discussed.

1. Introduction

In recent years, nanostructured elements such as carbon nanotubes, graphene sheets or nanoplates, nanobeams and nanomembranes have attracted a lot of attention from the research community. The reason behind such an interest is their superb mechanical and electronic properties which have then brought about their wide applicability in many emerging fields of nanotechnology [1–7]. Because of the increasingly applicability of the nanostructures in nanotechnology, fundamental knowledge of the mechanical behavior of them is desperately required.

When the dimensions of a nanostructure tend to very small scale, the size effect becomes pivotal. This is because that the structure at micron or sub-micron scales possesses discrete nature making the system become comparable to the interatomic or intermolecular spacing of that system [8]. Hence, the long range interatomic cohesive forces i.e., the small scale influences will affect the static and dynamic responses of the nanostructures and so, they must be regarded in the design of the structures at nanoscale. One of the most powerful tools which has provided valuable insights into various aspects of nanomaterials is the atomistic methods like molecular dynamic (MD) simulations. However, they require huge computational capacities to achieve accurate solution of the large-sized nanostructures [9]. In the presence of shortcomings of the atomic approaches, the attention is focused on the continuum mechanics models. The conventional continuum models lack the inclusion of small scale effects due to the neglect of the material microstructure at small size. On the path to the amendment of the classical continuum models,

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Eringen took a major step forward by proposing the nonlocal continuum theory [10,11]. This theory introduces atomic length scales directly into the constitutive relations as a material parameter. Ever since, researchers adopted the nonlocal continuum mechanics for bending, buckling and vibration analyses of the nanostructures [12–33].

Nanoplates are two-dimensional (2D) nanoscale structures possessing the brilliant physical and chemical properties and so, high potential applications in nano/micro electromechanical systems (NEMS/MEMS). Vibration behavior of these nanostructures needs to be well-known for the proper design of the nanodevices in NEMS/MEMS. In the open literature, different nonlocal plate models have been developed for the vibration analysis of the nanoplates [8,15–17,20,23,25,27,29,32–34]. Aghababaei and Reddy [16] reformulated the third-order shear deformation plate theory (TSDT) using the nonlocal elasticity theory of Eringen and presented the analytical solutions of bending and free vibration of a simply supported plate. Ansari et al. [20] developed a nonlocal plate model for free vibrations of single-layered graphene sheets. Murmu and Adhikari [23] employed the nonlocal theory of elasticity to study the vibration behavior of bonded double-nanoplate-systems. Malekzadeh and Shojaei [29] applied a two-variable refined plate theory including the small scale effects to the free vibration of the nanoplates. Ansari et al. [32] developed a nonlocal plate theory incorporating the interatomic potentials and predicted the biaxial buckling and vibration behavior of graphene. Using nonlocal theory, Hosseini-Hashemi and his co-workers [33] derived an exact solution for free vibration of FG circular/annular nanoplates based on the first order shear deformation plate theory (FSDT). Recently, Chakraverty and Behera [8] studied the vibration of isotropic rectangular nanoplates using nonlocal elasticity theory and classical plate theory (CPT).

In all the aforementioned works, the one or two-dimensional theories have been used to establish the size-dependent plate model. In the open literature, very few works exist on the three-dimensional modeling of the nanoscale plates. In this respect, Alibeigloo [35,36] analyzed the 3D free vibration of one- and multi-layered rectangular graphene sheets by using the theory of elasticity and state space approach. Alibeigloo and Pasha Zanoosi [37] analytically studied the bending of a simply supported rectangular nanoplate on the basis of the nonlocal continuum mechanics and the 3D theory of elasticity. On the other hands, the research works on the three-dimensional (3D) analyses of nanoplates are restricted to the all or two edges simply supported plates with no elastic foundations. Elastic foundations can be considered on the nanoplate to model the interaction between the elastic media and the plates in different engineering problems. Prompted by these voids, in this paper, an attempt is made to study the three-dimensional free vibration of FG nanoplates with arbitrary conditions resting on two-parameter elastic foundations. To this, the nonlocal constitutive equations of Eringen are incorporated into the three-dimensional equations of motion. To solve the problem, the GDQ method is used in all the coordinate directions enabling one to impose the top and bottom surface boundary conditions i.e., the foundation effects as well as any arbitrary condition at the other edges of the nanoplate. Some numerical results are presented and compared with the ones existing in the literature to validate the accuracy and convergence of the current approach. Afterward, a detailed parametric study is done to delineate the effects of boundary conditions and various model parameters on the natural frequencies of the FG nanoplates.

2. Nonlocal 3D elastic plate model for FG nanoplates

A FG nanoplate of length a, width b and thickness h supporting on Winkler–Pasternak elastic foundations at its bottom surface, as shown in Fig. 1, is considered. A Cartesian coordinate system located at the lowplane of the plate is used and the Pasternak model is employed to represent the reaction of the elastic foundation to the plate.

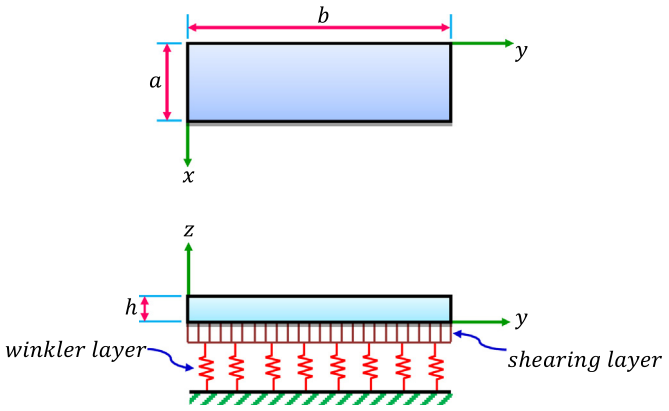

Fig. 1. A schematic of a FG nanoplate resting on elastic foundation with geometric parameters and coordinate systems.

2.1. Material properties of the FG nanoplate

The bottom surface $(z = 0)$ and the top surface $(z = h)$ of the plate are taken to be rich-metal and rich-ceramic, respectively. In the current study, the material properties of FGM are considered to be graded in the thickness direction and follow a power function of a spatial variable. Accordingly, the volume fractions of ceramic V^c and of metal V^m are stated in the following form:

$$V^c(z) = \left(\frac{z}{h}\right)^k, \quad V^m = 1 - V^c \tag{1}$$

where k denotes the volume fraction or material gradient index. Let C_{ij} and ρ be the material elastic coefficients and the mass density of the plate, respectively. Based on the rule of mixtures, the general form of these parameters is given by

$$C_{ij}(z) = C_{ij}^m V^m + C_{ij}^c V^c, \quad \rho(z) = \rho^m V^m + \rho^c V^c \tag{2}$$

From Eq. (1), in the case of $k=0$, the plate becomes fully ceramic and when $k \to \infty$, the plate reduces to a fully metal one. Also, the Poisson's ratio ν is considered to be constant.

2.2. Review of the nonlocal theory

The small-scale effects are taken into account by the nonlocal elasticity theory. The notion of this theory is that the stress at a point in an elastic continuum is dependent on the strain at all the points of the domain [10,11], whereas in the classical models the stress depends only on the strain of the same point. The simplified differential form of the nonlocal constitutive equation proposed by Eringen for a Hooken solid is expressed as [17]

$$\left(1 - \mu\nabla^2\right)\boldsymbol{\sigma} = \mathbf{t} \tag{3}$$

where $\mu=(e_0\hat{a})^2$ represents the nonlocal parameter or characteristic length leading to the inclusion of the size effects with e_0 as a constant appropriate to each material and $\hat{a}$ as the internal characteristic length (e.g., lattice parameter, C–C bond length and granular distance). ∇^2 is the Laplacian operator given by $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$. Also, $\mathbf{t}$ is the macroscopic stress tensor at a point which is related to strain by generalized Hooke's law as

$$\mathbf{t} = \boldsymbol{C}: \epsilon \tag{4}$$