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Nonlocal transient thermal analysis of a single-layered graphene sheet embedded in viscoelastic medium

Ashraf M. Zenkour [a,b,*]

[a] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[b] Department of Mathematics, Faculty of Science, Kafrelsheikh University, Kafrelsheikh 33516, Egypt

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ABSTRACT

The transient thermal analysis of a single-layered graphene sheet (SLGS) embedded in viscoelastic medium is presented by using the nonlocal elasticity theory. The elastic medium, which characterized by the linear Winkler's modulus and Pasternak's (shear) foundation modulus, is changed to a viscoelastic one by including the viscous damping term. The governing dynamical equation is obtained and solved for simply-supported SLGSs. Firstly; the effect of the nonlocal parameter is discussed carefully for the vibration and bending problems. Secondly, the effects of other parameter like aspect ratio, thickness-to-length ratio, Winkler-Pasternak's foundation, viscous damping coefficient on bending field quantities of the SLGSs are investigated in detail. The present results are compared with the corresponding available in the literature. Additional results for thermal local and nonlocal deflections and stresses are presented to investigate the thermal visco-Pasternak's parameters for future comparisons.

1. Introduction

The nanostructured materials such as single-layered (SL) and multi-layered (ML) graphene sheets are a relatively new class of materials and offer a variety of physical properties with many applications in several fields. Graphene is a new class of two-dimensional carbon nanostructure which holds great promise for the vast applications in many technological fields. The graphene sheet is a two-dimensional lattice structure and has many unique properties which cannot be matched by conventional materials. The GSs are mostly used in polymer composites as embedded structures to fortify them. Furthermore, the potential applications of the SLGSs as mass sensors and atomistic dust detectors have been investigated. Most of the studies on vibration, bending and buckling of nanoplates are carried out on SLGS and MLGS.

The notions of continuum mechanics have attracted a great deal of attention of many researchers to treat structures at the scale of nanometer. The classical continuum mechanics approaches are widely used but theory cannot predict the size effect. Successful applications of the classical continuum modeling to the bending response of nanostructures have been reported by a number of research workers [1–4]. However, the classical continuum mechanics is scale independent which makes its applicability to the small-scale nanomaterials somewhat questionable. The size effects are recognized to become more pronounced as the dimensions of nanostructures become very small. It has been suggested that the nonlocal continuum theory presented by Eringen [5–8] should be integrated in the continuum models for accurate prediction of nanostructures mechanical behaviors [9]. The nonlocal Eringen's theory is based on this assumption that the stress at a material point is considered as a function of the strain field at all material points in the continuum body. It is proposed for small scale problems like dislocations and cracks in materials, where stresses at a reference point are functions of the strains at all points of the body. The theory is found to be in good agreement with lattice dynamics model in studying plane waves and the experiment on phonon dispersion.

The extension of continuum mechanics to accommodate the size dependence of nanomaterials becomes another topic of major concern. Application of nonlocal continuum mechanics allowing for the small scale effects to vibration frequency analysis of nanomaterials has been also suggested by some other research workers in the study of nanostructures [10–23]. However, the inclusion of the viscous damping effect as a third foundation parameter is rare in the literature. Most SLGSs and DLGSs embedded on visco-Pasternak foundation are presented in the literature to investigate the nonlocal vibration frequencies [24–29]. In addition, the inclusion of the thermal field in the problem of GSs embedded in elastic medium is also discussed. Ansari et al. [30] have studied the axial buckling characteristics of SWCNTs including thermal environment effect. Satish et al. [31] have presented the thermal

* Corresponding author at: Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia.
E-mail address: zenkour@kau.edu.sa

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vibration analysis of orthotropic nanoplates using two-variable refined plate theory and nonlocal continuum mechanics for small scale effects. Liu et al. [32] have studied the thermo-electro-mechanical free vibration of piezoelectric nanoplates based the nonlocal theory and the classical Kirchhoff's theory. Xu et al. [33] have investigated the nonlinear bending behavior of a bilayer (double-layered plate) rectangular graphene sheet subjected to a transverse uniform load in thermal environments. Mohammadi et al. [34] have studied the buckling behavior of orthotropic SLGSs in thermal environment by using nonlocal elasticity theory. Nami et al. [35] have used the nonlocal elasticity theory and third-order shear deformation theory to investigate the thermal buckling analysis of functionally graded rectangular nanoplates. Zhang et al. [36] have investigated the transient analysis of SLGSs by using the element-free kp-Ritz method.

The transient thermal analysis of a SLGS embedded in a visco-Pasternak's (three-parameter) medium is presented to display various characters. The governing equation is obtained and solved analytically for a simply-supported SLGS. The effects of different parameters on the natural vibration frequencies, deflections and stresses are investigated. Sample results are tabulated and plotted for sensing the effect of all used parameters and to investigate the nonlocal and visco-Pasternak's parameters for future comparisons.

2. Basic equation of single-layered graphene sheet (SLGS)

Let us consider a single-layered graphene sheet (SLGS) of length l, width b and uniform thickness h as shown in Fig. 1. The SLGS is made of a homogeneous isotropic and linearly elastic material with Young's modulus E, Poisson's ratio ν, shear modulus G and material density ρ.

2.1. The visco-Winkler-Pasternak foundations

The two-parameter Pasternak's model is the most natural extension to the one-parameter Winkler's model. It considers a shear interaction between the spring elements by connecting the ends of the springs to a plate of an incompressible shear layer. The present SLGS is embedded in a homogeneous three-parameter viscoelastic medium. The foundation model is characterized by the linear Winkler's modulus K_1, the Pasternak's (shear) foundation modulus K_2, and the damping coefficient C_t of the viscoelastic medium. Taking into account the un-bonded contact between the SLGS and medium, the interaction follows the three-parameter visco-Pasternak-type foundation model as

$$R_f=\left(K_1-K_2 l^2\nabla^2+C_t\frac{\partial}{\partial t}\right)w, \tag{1}$$

where w is the transverse displacement and ∇^2 is the Laplacian (second-order spatial gradient). Here, we have introduced the SLGS length l in Eq. (1) for maintaining the dimension of K_1 and K_2 to be the same. If the foundation is modelled as the visco-Winkler foundation, the coefficient K_2 in Eq. (1) is zero. The viscosity term may be omitted by setting C_t=0 to get the analysis of the SLGS embedded in pure elastic medium.

2.2. Nonlocal classical plate theory

The most general form of the constitutive relation in nonlocal elasticity theory involves an integral over the entire region of interest. The integral contains a nonlocal kernel function, which describes the relative influence of the strains at the various locations of the body on the stress at the material point under consideration. Specifically, the constitutive equation of nonlocal elasticity for homogenous and isotropic elastic solids read

$$\sigma_{kl}(x) = \int_V \psi(|x-x'|)\tau_{kl}(x')\mathrm{d}V(x') = 0 \tag{2}$$

where σ_{ij} is the nonlocal stress tensor, V is the volume occupied by the elastic body, $|x-x'|$ denotes distance in Euclidean space, and the nonlocal kernel $\psi(|x-x'|)$ accounts for the effect of the strain at the point x' on the stress at the point x in the elastic body. The parameter ψ is an internal characteristic length (e.g., lattice parameter, granular distance, length of C–C bonds).

The quantity $\tau_{kl}(x')$ denotes the local stress tensor for which the standard local constitutive equation is adopted, i.e.

$$\tau_{kl}(x')=\lambda\varepsilon_{mm}(x')\delta_{kl}+2\mu\varepsilon_{kl}(x')-\gamma\delta_{kl}T, \tag{3}$$

where $\varepsilon_{kl}(x')$ is the classical local strain tensor at x'. Here, $\Delta T=T-T_0$ denotes the increment temperature in which T is the thermodynamical temperature and T_0 is the reference temperature, λ and μ being Lamé's constants, $\gamma=\alpha(3\lambda+2\mu)=\alpha E/(1-\nu)$ is the coupling parameter, α represents the coefficient of thermal expansion for the SLGS.

The small strain-displacement relations are given by the usual relations

$$\varepsilon_{kl}(x') = \frac{1}{2}\left(\frac{\partial u_k(x')}{\partial x'_l}+\frac{\partial u_l(x')}{\partial x'_k}\right) \tag{4}$$

where $u_l(x')$ is the displacement vector at a reference point x' in the body. For an appropriate form of the nonlocal kernel [5–8], it turns out that the nonlocal internal constitutive relation given by Eq. (1) can be inverted to yield the following pseudo-local

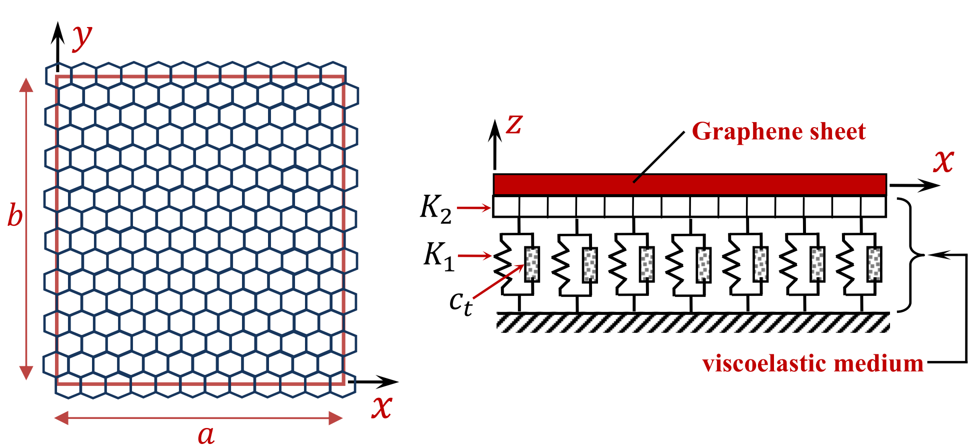

Fig. 1. A continuum plate model of a single-layered graphene sheet embedded in a viscoelastic medium.