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Negative differential conductivity in quantum well with complex potential profile for electron–phonon scattering

S.R. Figarova [a], G.N. Hasiyeva [a], V.R. Figarov [b,*]

[a] Department of Physics, Baku State University, 23 Z.Khalilov st., AZ 1148, Baku, Azerbaijan
[b] Institute of Physics, 131 H.Javid ave., AZ 1143, Baku, Azerbaijan

HIGHLIGHTS

- Negative differential conductivity.
- Quantum well.
- Complicated potential profile.

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ABSTRACT

The effect of phonon scattering on electrical conductivity (EC) of 2D electron gas in quantum well (QW) systems with a complicated potential profile is described. Dependence of QW electrical conductivity on QW parameters (such as QW width, Fermi level positions etc.) when phonon scattering is employed has been calculated. NDC in EC when it varies with width of the QW has been found.

1. Introduction

Structures with different shape quantum wells (QWs), within which charge carrier motion is restricted, are widely utilized in nanoelectronics. Transport phenomena in QWs are substantially distinguished from those in bulk and possess a series of features. Among them are negative differential conductivity (NDC), electrical conductivity (EC) oscillations, semimetal–semiconductor transition and high mobility. These effects are explained by properties inherit in low-dimensional systems, depend on QW profile and sizes and scattering mechanisms in low-dimensional systems. NDC in 2D systems has been attributed to dynamical localization at the expense of electron Bloch oscillations, presence of strong electric fields, non monotonic drift mobility, tunneling, scattering mechanism features in low-dimensional systems, doping, and also to spatial transport of electrons from QW into parallel layer conduction [1] and scattering-induced NDC [2]. The latter exhibits in *GaAs* QWs at high doping of structures (10^{23} – 10^{24} M^{-3}) [3]. NDC magnitude is governed by the density of states and QW shape [4]. NDC in QW is experimentally observed solely in some special cases [5–7]. Mainly, when investigated QWs, infinitely deep rectangular (most common), triangle and parabolic, and also δ- potential QW models are taken. However, the real potential QW is of more complex shape, neither infinite nor flat. On the other hand, at present epitaxial growth technique allows one to fabricate structures with arbitrary QW potential profiles. From this standpoint, theoretical studies of phenomena in QW structures with complicated potential profile are of interest.

In the paper we consider 2D electron gas EC in QW with complicated potential profile. An influence of QW parameters on EC for phonon scattering is investigated. We obtain that EC is non monotonic with potential QW width and height, and for acoustic phonon scattering depending on the relationship between the Fermi level and QW parameters becomes negative. The oscillation

* Corresponding author.
E-mail addresses: figarov@bsu.az (S.R. Figarova), Xasieyva.guler@mail.ru (G.N. Hasiyeva), figarov@physics.ab.az (V.R. Figarov).

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period dependent on the charge carrier concentration is determined.

2. General form for EC in complicated potential profile QW

As mentioned, the chief model of rectangular QW is a crude approximation: in real structures spatially separated charges cause an appearance of electrostatic potential, this in turn results in band edges bending, transforming the rectangular QW into the parabolic type one [8]. In order to find energy levels for the potential profile having a finite width but no angles, we employ the function [9]:

$$U(z) = \frac{U_0}{\cos^2(z/a)} \tag{1}$$

here a is the QW width, U_0 is the potential energy minimum. QW potential (1) is illustrated schematically in the insert in Fig. 1. It describes wide and narrow QWs, includes rectangular and parabolic potentials, its profile depends on the QW width and Fermi level position.

Solving the Schrödinger equation with potential energy (1) gives energy spectrum of 2D electron gas:

$$\varepsilon_{n,k_x,k_y} = \frac{\hbar^2 k_\perp^2}{2m} + \varepsilon_n \tag{2}$$

where $k_\perp^2 = k_x^2 + k_y^2$, m is the electron effective mass and ε_n acquires the form:

$$\varepsilon_n = \varepsilon_0\left(1 + 2n + \sqrt{1 + \frac{U_0}{\varepsilon_0}}\right)^2 \tag{3}$$

here $\varepsilon_0 = \hbar^2\pi^2/8ma^2$ at $n = 0$ and $U_0 = 0$, n=0,1,2,… is the quantum number. Models of rectangular potential and parabolic QWs are particular cases of energy spectrum (3).

For energy spectrum (3), the density of states of 2 DEG is

$$g(\varepsilon) = \frac{m}{\pi\hbar^2}\sum_n \Theta(\varepsilon - \varepsilon_n) \tag{4}$$

where $\Theta(\varepsilon - \varepsilon_n)$ is the Heaviside function.

As seen from formulae (2) and (3), electrons in conduction band move practically freely in the plane parallel to QW boundaries; the transverse electron motion turns to be quantized, but electrons at these discrete levels retain freedom of motion in two other directions. To describe such a transport process, the semi-classical approach is entirely grounded.

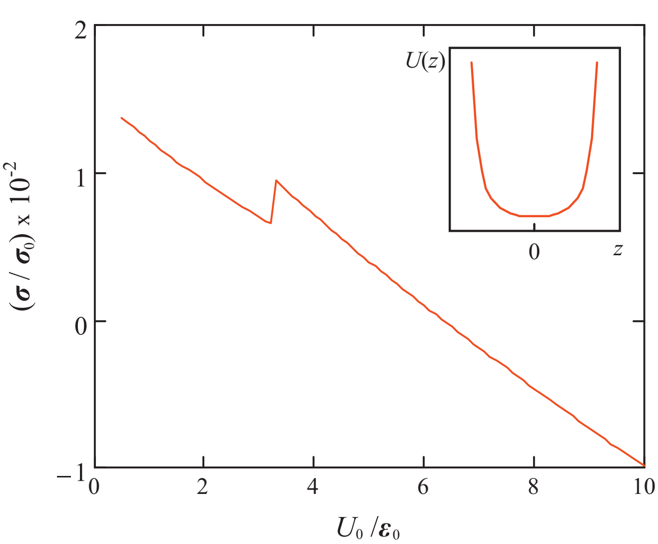

Fig. 1. Relative EC σ/σ_0 vs. the QW parameter U_0/ε_0 for acoustic phonon scattering.

In the geometry of the problem, $\vec{E}(E_x, 0, 0)$, current density is defined as [10]

$$j_x = -\frac{2e^2}{V}\sum_{n,k_x,k_y}\left(\frac{\partial f_0}{\partial\varepsilon}\right)\tau(\vec{k})v_x^2 E_x \tag{5}$$

where f_0 is the Fermi–Dirac equilibrium distribution function, $\tau(\vec{k})$ is the relaxation time.

Thence for EC, we get:

$$\sigma = e^2\sum_x \tau\left(\vec{k}\right)\left(-\frac{\partial f_0}{\partial\varepsilon}\right)v_x^2 \tag{6}$$

From (6) passing from summation over k_x and k_y to integration in polar coordinates $d\vec{k}_\perp = k_\perp dk_\perp d\phi$, we find:

$$\sigma = e^2 n_{el}\langle\tau/m\rangle \tag{7}$$

The angular brackets $\langle\ldots\rangle$ denote

$$\langle\tau/m\rangle = \frac{m}{\pi a n_{II}\hbar^2}\sum_n\int_{\varepsilon_n}^{\infty}\Theta(\varepsilon - \varepsilon_n)(\varepsilon - \varepsilon_n)\frac{\tau}{m}\left(-\frac{\partial f_0}{\partial\varepsilon}\right)d\varepsilon \tag{8}$$

$$n_{el} = \frac{m}{\pi\hbar^2}\sum_n\int_{\varepsilon_n}^{\infty}\Theta(\varepsilon - \varepsilon_n)(\varepsilon - \varepsilon_n)\left(-\frac{\partial f_0}{\partial\varepsilon}\right)d\varepsilon \tag{9}$$

Formulae (7)–(9) are just for any dependence ε_n on the quantum number n, i.e. for any shape of the QW. To derive an analytical expression EC for specified QW (3) we need in an explicit relaxation time. In view of $\tau^{-1}\sim W(\varepsilon)g(\varepsilon)$, where $W(\varepsilon)$ is the scattering probability [10] for an electron–phonon scattering in 2D systems it can be written [11–12]:

$$\tau = \frac{1}{g(\varepsilon)}\left(\frac{\varepsilon_\perp}{k_0T}\right)^r\left(\frac{2mk_0T}{\hbar^2}\right)^r\frac{1}{A_r} \tag{10}$$

here $g(\varepsilon)$ is given by formula (4), for scattering by acoustic phonons $r = 0$, $A_0 = \frac{\pi E_1^2 k_0 T}{\hbar\rho v_0^2}$ (E_1 is the deformation potential), for scattering by optical phonons $r = 1$, $A_1 = \frac{2\pi^2 e^2 k_0 T}{\hbar\chi^*}\left(\frac{1}{\chi^*} = \frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right)$, χ_∞ and χ_0 are high-frequency and static dielectric permeability of the crystal).

Having put (10) and (4) into (7) for EC we receive:

$$\sigma = \frac{e^2\tau_0}{m}n_0\frac{\sum_n\int_{\varepsilon_n}^{\infty}\Theta(\varepsilon^* - \varepsilon_n^*)(\varepsilon^* - \varepsilon_n^*)^{r+1}\left(-\frac{\partial f_0}{\partial\varepsilon^*}\right)d\varepsilon^*}{\sum_n\Theta(\varepsilon^* - \varepsilon_n^*)} \tag{11}$$

where $n_0 = \frac{mk_0T}{\pi a\hbar^2}$, $\tau_0 = \frac{\pi\hbar^2 a}{m}\left(\frac{2mk_0T}{\hbar^2}\right)^r\frac{1}{A_r}$, $\varepsilon^* = \frac{\varepsilon}{k_0T}$, $\varepsilon_n^* = \frac{\varepsilon_n}{k_0T}$.

In the case of a degenerate electron gas from (11) we have σ:

$$\sigma = \frac{e^2\tau_0}{m}n_0\frac{\sum_n\Theta(\varepsilon_F^* - \varepsilon_n^*)(\varepsilon_F^* - \varepsilon_n^*)}{\sum_n\Theta(\varepsilon_F^* - \varepsilon_n^*)} \tag{12}$$

where $\varepsilon_F^* = \frac{\varepsilon_F}{k_0T}$ and the Fermi level ε_F is [13]

$$\varepsilon_F = \frac{n_{el}\pi\hbar^2}{m(\bar{n}+1)} + U_0 + 2\sqrt{\varepsilon_0^2 + \varepsilon_0 U_0}\left(\bar{n}+1\right) + \frac{4\varepsilon_0\left(\bar{n}^3 + 3\bar{n}^2 + 2\bar{n} + 1.5\right)}{3(\bar{n}+1)} \tag{13}$$

where $\bar{n}$is the integer part of the number $n = \sqrt{2m\varepsilon_F}\frac{a}{\pi\hbar} - \frac{1}{2} - \frac{1}{2}\sqrt{1 + \frac{U_0}{\varepsilon_0}}$ which is found from the condition $\varepsilon_F = \varepsilon_n$.

Substituting (13) into (12) and performing summation over n yields for relative EC: