Physica E 75 (2016) 15–21

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Finite-frequency noise in a topological superconducting wire

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HIGHLIGHTS

- Finite-frequency current correlations are studied for a superconducting wire.
- Majorana bound states are allowed at the ends of the wire.
- The wire is attached to two terminals at one of its ends.
- In the topological phase we find vanishing cross-correlations for large frequency.

ARTICLE INFO

Article history:
Received 30 June 2015
Received in revised form
6 August 2015
Accepted 16 August 2015
Available online 18 August 2015

Keywords:
Quantum electron transport
Hybrid superconducting systems
Topological superconductors
Current-current correlations

ABSTRACT

In this paper we study the finite-frequency current cross-correlations for a topological superconducting nanowire attached to two terminals at one of its ends. Using an analytic 1D model we show that the presence of a Majorana bound state yields vanishing cross-correlations for frequencies larger than twice the applied transport voltage, in contrast to what is found for a zero-energy ordinary Andreev bound state. Zero cross-correlations at high frequency have been confirmed using a more realistic tight-binding model for finite-width topological superconducting nanowires. Finite-temperature effects have also been investigated.

1. Introduction

One of the prototypical systems which host Majorana Bound States (MBS) is the Kitaev chain [1], a discrete model for a one-dimensional p-wave superconductor. Such a model can be realised in a semiconducting nanowire with strong spin-orbit coupling by placing it in close proximity to a s-wave superconductor, thus inducing superconductivity in the wire, and applying a strong magnetic field which leads to a large Zeeman splitting [2,3]. With possible solid-state realisations available, several experimental studies have gathered evidence compatible with the existence of MBSs [4–8]. As a result, the quest for an unambiguous signature of the presence of a MBS has become a priority and is stimulating a significant research effort [9–13]. Up to now only a few papers have focused on the consequences of MBSs on the behaviour of current correlations [14–24]. Very recently Haim et al. (Ref. [25]) have considered the spin-resolved current cross-correlations, finding that they are negative for a MBS in the case of correlations between opposite spins.

In this paper, we consider a semi-infinite topological superconducting wire attached to two normal terminals at one end, as shown in Fig. 1. A bias voltage V is applied to the two normal contacts (labelled 1 and 2), while the superconducting wire is grounded. We calculate the cross-correlations at finite frequency between the currents I_1 and I_2 flowing in the two normal leads. Our main finding is that the cross-correlations at frequencies larger than twice the voltage V vanish at zero temperature, when the superconducting wire is in the topological phase. On the contrary, in the presence of an ordinary zero-energy Andreev Bound State (ABS), which similarly to a MBS gives rise to a zero-bias peak in the differential conductance, the cross-correlations at high frequency are in general non-zero and depend on the details of the system. The origin of this phenomenon can be attributed to the peculiar structure of the energy-dependent scattering matrix for reflections off a MBS. In addition, at zero-frequency the cross-correlations are

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http://dx.doi.org/10.1016/j.physe.2015.08.028
1386-9477/

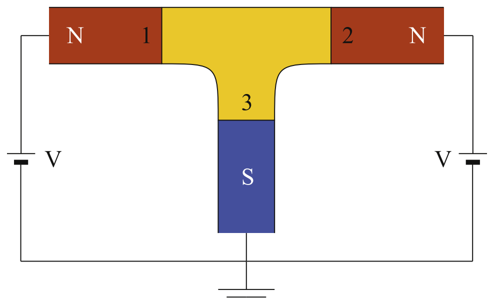

Fig. 1. Schematic setup of the system. A superconducting (S) nanowire, which can be driven in and out of the topological phase by an applied magnetic field, is contacted by means of a beam splitter to two normal (N) leads, labelled 1 and 2.

always negative in the presence of a MBS, analogously to the case of spin-resolved spin-up/spin-down cross-correlations [25], but may be positive in the case of an ABS. These results have been obtained with a simple analytical model whereby two normal leads are coupled either to a Majorana state or a zero-energy level. Zero cross-correlations at high frequency are then confirmed using a more realistic tight-binding model based on the semiconducting-nanowire realisation of a one-dimensional *p*-wave superconducting wire.

2. Formalism and methods

In this section we briefly review the scattering approach for the finite-frequency current-current correlations in hybrid superconducting systems [26]. The (non-symmetrized) current-current correlator between lead i and i' is defined as

$$S_{ii'}(t) = \langle I_i(t) I_{i'}(0)\rangle - \langle I_i\rangle\langle I_{i'}\rangle, \tag{1}$$

where $I_i(t)$ is the current[1] operator at time t relative to terminal i and $\langle\cdots\rangle$ stands for the quantum-statistical average. Taking the Fourier transform of $S_{ii'}(t)$ one obtains the finite-frequency correlator as

$$S_{ii'}(\omega) = \int S_{ii'}(t)\, e^{i\omega t}\, dt. \tag{2}$$

Within the Landauer-Büttiker scattering approach [28,30], the finite-frequency current-current correlator in a hybrid superconducting system, calculated in the normal leads, is given by [26]

$$S_{ii'}(\omega) = \frac{e^2}{2h} \sum_{\substack{\alpha,\alpha'\\ \beta,\beta'}} \sum_{\substack{\sigma,\sigma'\\ \tau,\tau'}} \sum_{j,j'} \mathrm{sign}(\beta)\mathrm{sign}(\beta') \times \int_{-\infty}^{+\infty} dE\, A^{\alpha j\sigma}_{\alpha' j'\sigma'}(\beta, E, E+\hbar\omega, \tau, i) \times A^{\alpha' j'\sigma'}_{\alpha j\sigma}(\beta', E+\hbar\omega, E, \tau', i') \times f_{\alpha i}(E)[1 - f_{\alpha' i'}(E+\hbar\omega)], \tag{3}$$

where the indices $\alpha, \alpha', \beta, \beta' \in \{\pm 1\}$ indicate electrons $(+1)$ and holes (-1) in Nambu space, $\sigma, \sigma', \tau, \tau'$ refer to the spin-projection quantum number and i, i', j, j' label the leads. The Fermi distribution function in the normal lead i for a α-like particle at temperature T and voltage V_i is given by

$$f_{\alpha i}(E) = \left[1 + \exp\left(\frac{E - \alpha e V_i}{k_B T}\right)\right]^{-1}. \tag{4}$$

Moreover, in Eq. (3) we have defined

$$A^{\alpha j\sigma}_{\alpha' j'\sigma'}(\beta, E, E', \tau, i) = \delta_{\alpha\beta}\delta_{\sigma\tau}\delta_{ji}\delta_{\alpha'\beta}\delta_{\sigma'\tau}\delta_{j'i} - [s^{i\tau j\sigma}_{\beta\alpha}(E)]^{\star} s^{i\tau j'\sigma'}_{\beta\alpha'}(E'), \tag{5}$$

where $s^{i\tau j\sigma}_{\beta\alpha}(E)$ is the scattering amplitude at energy E for a α-like particle with spin σ injected from lead j to be reflected as a β-like particle with spin τ in lead i. In the rest of this paper, we shall focus on the symmetrized noise, defined as $S^S_{ii'}(\omega) = S_{ii'}(\omega) + S_{ii'}(-\omega)$, since this is the quantity that is measured by a classical detector [31]. We shall furthermore assume that the two normal terminals are kept at the same voltage ($V_1 = V_2 = V$).

3. Analytic 1D model

The system depicted in Fig. 1 can be modelled in a simple way by composing [30] the scattering matrix s^{M} of a normal lead coupled to a Majorana state with the scattering matrix s^{bs} of a 3-leg beam splitter, which describes the connection to terminals 1 and 2. The matrix s^{M} can be calculated from the Hamiltonian describing a normal lead coupled to a Majorana state (see Ref. [25])

$$H = \sum_{k,\sigma} \epsilon_k \psi^{\dagger}_{k\sigma}\psi_{k\sigma} + i\gamma \sum_{k,\sigma} (t_\sigma \psi_{k\sigma} + t_\sigma \psi^{\dagger}_{k\sigma}), \tag{6}$$

where the first term describes the lead, $\psi^{\dagger}_{k\sigma}$ being the creation operator of a spin-σ particle with momentum k and energy ϵ_k, and the second term the coupling to the localised Majorana fermion γ. Without loss of generality, we assume the coupling parameters $t_\uparrow$ and $t_\downarrow$ to be real. Using Eq. (6) the scattering matrix in Nambu space takes the following form

$$s^{\mathrm{M}} = \begin{pmatrix} r^{\mathrm{M}}_{ee} & r^{\mathrm{M}}_{eh} \\ r^{\mathrm{M}}_{he} & r^{\mathrm{M}}_{hh} \end{pmatrix}. \tag{7}$$

Here $r^{\mathrm{M}}_{\alpha\beta}$ are matrices, in spin space, of reflection amplitudes given by

$$r^{\mathrm{M}}_{ee} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{iE - \Gamma}\begin{pmatrix} \Gamma_\uparrow & \sqrt{\Gamma_\uparrow \Gamma_\downarrow} \\ \sqrt{\Gamma_\uparrow \Gamma_\downarrow} & \Gamma_\downarrow \end{pmatrix} \tag{8}$$

and

$$r^{\mathrm{M}}_{he} = \frac{1}{iE - \Gamma}\begin{pmatrix} \Gamma_\uparrow & \sqrt{\Gamma_\uparrow \Gamma_\downarrow} \\ \sqrt{\Gamma_\uparrow \Gamma_\downarrow} & \Gamma_\downarrow \end{pmatrix}, \tag{9}$$

where $\Gamma_\uparrow = 2\pi\nu_0 |t_\uparrow|^2$, $\Gamma_\downarrow = 2\pi\nu_0 |t_\downarrow|^2$, $\Gamma = \Gamma_\uparrow + \Gamma_\downarrow$ with ν_0 being the density of states of the normal leads, while r^{M}_{eh} and r^{M}_{hh} are determined by particle-hole symmetry $r^{\mathrm{M}}_{\alpha,\beta}(E) = [r^{\mathrm{M}}_{-\alpha,-\beta}(-E)]^{\star}$.

Assuming no spin mixing to occur in the beam-splitter, the block of the scattering matrix s^{bs} for spin-σ electrons is a 3×3 unitary matrix which can be parameterized as follows [32]

$$s^{\mathrm{bs}}_{e\sigma} = \begin{pmatrix} c_\sigma & \lambda_{1\sigma} & \lambda_{2\sigma} \\ \lambda_{1\sigma} & a_{1\sigma} & b_\sigma \\ \lambda_{2\sigma} & b_\sigma & a_{2\sigma} \end{pmatrix}. \tag{10}$$

Here $\lambda_{1(2)\sigma}$ is the scattering amplitude for an electron with spin σ injected from lead 1(2) to be transmitted in 3 (with $\lambda^2_{1\sigma} + \lambda^2_{2\sigma} \le 1$),

[1] In this paper we consider only quasiparticles current and neglect the role of displacement currents; the latter might induce corrections to the noise at high frequency in the case of strongly energy-dependent density of state [27–29]. This must be taken into account in the analysis of actual experimental data. Such corrections, however, depend on the details of the system and their discussion is beyond the scope of the present paper.