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Analytical modeling for the determination of nonlocal resonance frequencies of perforated nanobeams subjected to temperature-induced loads

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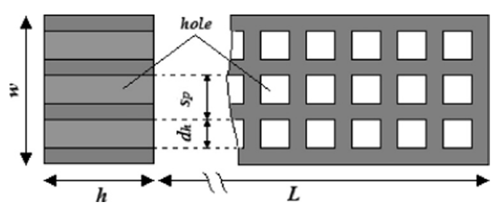
HIGHLIGHTS

- Resonance frequencies of perforated nanobeams depends on size and of holes.
- Thermal leads and small scale effects decrease resonance frequencies.
- Thermal and nonlocal frequency ratios varies as the resonance frequency change.

GRAPHICAL ABSTRACT

Geometry of perforated nanobeam structure with periodic square holes network. Part of structure is cut away for clarity. Small scale and thermal loads effects are considered in the modeling.



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ABSTRACT

This paper is concerned with the investigation of thermal loads and small scale effects on free dynamics vibration of slender simply-supported nanobeams perforated with periodic square holes network and subjected to temperature-induced loads. The Euler–Bernoulli beam model (EBM) and shear beam model (SBM) developed for the determination of resonance frequency are derived by modifying the standard Timoshenko beam equations. The small scale effect is included by using the Eringen's nonlocal elasticity theory while the thermal loads effect is included by considering the additional axial thermal force in the standard differential equations. Numerical results are shown that the resonance frequency change, the thermal loads and the small scale effects are depended on size and number of holes. Thus, numerical results are discussed in detail for a properly investigation of the dynamic behavior of perforated nanobeams which are of interest in the development of resonant devices integrated in micro/nanoelectromichanical systems (M(N)EMS).

1. Introduction

Recently, the dynamic vibrations analysis of nanostructures has received considerable attention and has been an active area of scientific researches. Based on the classical elasticity theory which is a scale independent theory [1,2] and the Timoshenko beam theory which covers bending moment, rotary inertia and shear distortion effects [3–7], various analytical beam models have been derived for buckling and vibrations. Simply supported beams are typical and simple structures frequently encountered in engineering applications for theoretical analysis on small deflections [5]. Several authors have considered additional effects such as of surface stress [8], piezoelectric parameters [9], thermal loads [10] and axial compression [11] to investigate the dynamic behavior of

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simply supported nanobeams integrated in micro/nano-electromichanical systems (M/NEMS). In their contributions, the authors oriented to analyze the resonance frequency change by varying nanostructures dimensions size and additional effect conditions. However, using the classical elasticity theory of materials, the investigation is unable to predict frequency change due to the presence of small scale effects at nanometric dimensions. Indeed, numerous experiments and numerical simulations have demonstrated the size-dependent deformation behavior of nanobeams [12–17]. Unlike the classical elasticity theory, nonlocal elasticity theory of Eringin [18–20] is a scale dependent theory accounts the nonlocal effects on resonance frequencies of simply supported beams at nanometric dimensions [21–24]. Nowadays, researchers have applied nonlocal elasticity theory for different materials [25–30] to predict linear and nonlinear [1,31] resonance frequency for uniform and nonuniform [32–35] nanostructures.

Perforated nanobeam is a common component largely used in advanced technologies especially for optomechanics and photonics [36–40]. However, the dynamic vibrations analysis of perforated nanobeams has not been studied as extensively as full nanobeams despite their importance in advanced technologies. Indeed, in the above studies, the effect of holes on dynamic vibrations has been investigated only for specific cases and applications due to the complication in the problem formulation. For example, Sharpe et al. [41] have investigated the effect of holes on the mechanical properties of polysilicon thin film. Their results showed that the Young's modulus value decrease of 12% and the strength of the holed specimens drops by 50%due to the presence of holes. Rabinovich et al. [42] have investigated the effect of holes on the Young's modulus and shear modulus by study the electromechanical behavior of a perforated membrane. They have showed that Young's modulus and shear modulus are strongly affected by holes with a decrease of 24% and 30% respectively.

In the last few years, the study of the effect of holes is concerned with the viscous losses around plates for the squeeze damping [43,44], the anchor losses in bulk plate resonators [45] and the resonance frequencies change for perforated beams [46]. This latter study may be considered as aclearly and very significant contribution in which *Luschi* and *Pieri* have developed analytical expressions for the equivalent bending stiffness and shear stiffness for perforated beam structures with periodic square holes network, and they have determined the resonance frequencies expression.

In this work, we perform calculations of the small scale and the thermal loads effects on the resonance frequency of a simply supported perforated nanobeam with an axial compression [21,25,34] for varying numbers and sizes of holes. The paper is organized as follows: in Section 2, the resonance frequency models are developed by using Eringen's nonlocal elasticity and Timoshenko beam theories. The small scale and thermal loads effects are investigated with calculations and discussions in Section 3. Concluding remarks are given in Section 4.

2. Problem formulation

In this section, the dynamics behavior of perforated nanobeam will be analyzed by using the Eringen's nonlocal elasticity theory and the Timoshenko beam theory containing the equivalent characteristic parameters for bending and shear stiffness developed by Luschi and Pieri [46]. To this purpose, we consider a nanobeam of length L, width $b$and thickness h, with periodic square holes network of spatial period s_P and size of hole d_h. We also define N as the number of holes along the section, and $\beta=(d_h/s_P)$as the hole size ratio which can range from 0 (full beam) to 1. Fig. 1 presents the geometry of perforated nanobeam structure with

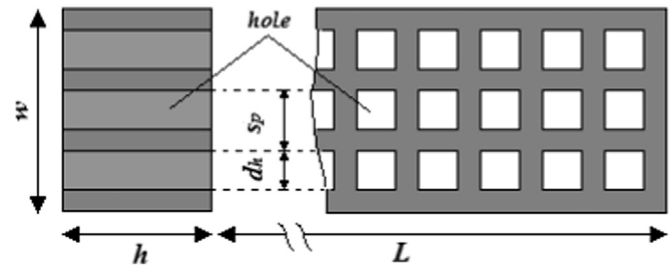

Fig. 1. Geometry shape of perforated nanobeamstructure with periodic square holesnetwork. Part of the total length of structure is cut away for clarity.

periodic square holes network proposed in this paper.

Unlike the Euler–Bernoulli beam theory which takes in consideration the bending effect, Timoshenko [3] proposed a beam theory which adds the shear effect as well as the rotation effect to the bending effect [4,46]. By using the Timoshenko beam theory, the standard equations for the dynamic vibration of a perforated nanobeam subjected to temperature-induced loads can be given by two coupled differential equations expressed in terms of the flexural deflection $w$and the rotation angle ψ of the cross-section. The coupled equations have been formulated as [47–49]:

$$\frac{\partial V}{\partial x} - \rho A_{eq}\frac{\partial^2 w}{\partial t^2} + F_a\frac{\partial^2 w}{\partial x^2} = 0 \tag{1}$$

$$\frac{\partial M}{\partial x} + V = \rho I_{eq}\frac{\partial^2 \psi}{\partial x^2} \tag{2}$$

where M and V are the nonlocal bending moment and the nonlocal shear force respectively; ρA_{eq} is the equivalent mass per unit length and ρI_{eq} is the equivalent rotational inertia per unit length. $F_a=F_t$-F_c is the result of the axial compressive force F_c [21,25] and the axial thermal force F_t which depends on the temperature change T and the thermal expansion coefficient θ of the beam material. The nonlocal bending moment M, the nonlocal shear force V and the additional axial force F_t are defined by Eqs. (3)–(5) respectively:

$$M = \mu^2\frac{\partial^2 M}{\partial x^2} + EI_{eq}\frac{\partial \psi}{\partial x} \tag{3}$$

$$V = \mu^2\frac{\partial^2 V}{\partial x^2} + kAG_{eq}\left(\frac{\partial w}{\partial x} - \psi\right) \tag{4}$$

$$F_t = \left(\frac{EI_{eq}\rho A_{eq}}{\rho I_{eq}}\right)\theta T \tag{5}$$

where EI_{eq} is the equivalent bending stiffness, GA_{eq} is the equivalent shear stiffness, $k$is the shear factor equal 5/6 for a rectangular cross-section [22] and $\mu=\mu^* L$ is the nonlocal parameter revealing the small-scale effect. Luschi and Pieri [47] determined the analytical expressions for the equivalent parameters ρA_{eq}, ρI_{eq}, EI_{eq} and GA_{eq} as functions of the number of holes N and the filling ratio $\alpha=(1-\beta)$.The analytical expressions of ρA_{eq}, ρI_{eq}, EI_{eq} and GA_{eq} have been determined by using Eqs. (6)–(9) respectively [46]:

$$EI_{eq} = \frac{EI_{eb}(N+1)\,\alpha\,\left(N^2+2N+\alpha^2\right)}{\left(1-\alpha^2+\alpha^3\right)N^3+3\alpha N^2+\left(3+2\alpha-3\alpha^2+\alpha^3\right)\alpha^2 N+\alpha^3} \tag{6}$$