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An improved model for the cantilever NEMS actuator including the surface energy, fringing field and Casimir effects

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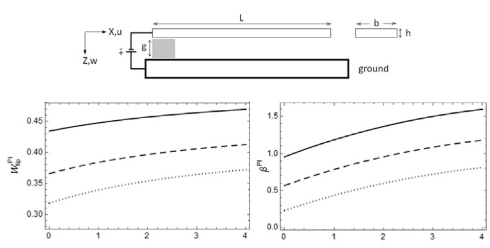
HIGHLIGHTS

- By considering the modified boundary conditions, the instability of a NEMS electrostatic actuator in precense of Casimir force, and surface energy, is extensively studied.
- The potential energy principle is applied in order to satisfy the equilibrium state in structures subjected to static loading.
- To solve the nonlinear fourth-order boundary value problem, the Duan–Rach method of determined coefficients (MDC) is applied.

GRAPHICAL ABSTRACT

The pull-in instability of a NEMS device incorporating the electrostatic force and Casimir intermolecular attraction for different values of the surface parameter with modified boundary conditions will be investigated.



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ABSTRACT

The influence of the surface energy on the instability of nano-structures under the electrostatic force has been investigated in recent years by different researchers. It appears that in all prior research, the response of all structures becomes softer due to the surface effects. In the present study, the pull-in instability of a NEMS device incorporating the electrostatic force and Casimir intermolecular attraction for different values of the surface parameter is investigated by the Duan–Rach method of determined coefficients (MDC) in order to identify the remarkable effect of the surface energy. Although the obtained results verify the behavior of such structures in presence of the fringing field and the Casimir attraction same as the previous investigations, however the incremental effects of the surface energy cause the aforementioned structures to behave more stiffly in contrast.

1. Introduction

Determination of the pull-in parameters, i.e. the pull-in voltage and deflection, is of the primary importance in the design procedure of micro-electromechanical system (MEMS) and nano-electromechanical system (NEMS) devices. Micro- and nano- actuators are often composed of two conductive electrodes in which at least one of them is movable. The movable electrode deflects towards the other one and becomes unstable at a critical voltage, namely the pull-in voltage. Kuang and Chen [1] and Lin and Zhao [2] studied the pull-in behavior of micro-structures.

Emerging new physical effects through a scale change in geometry from micro to nano may cause some additional parameters such as intermolecular attractions and surface effects into the

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theory of nano-scale structures which can be neglected sometimes in the micro-structures.

A remarkable number of studies have been performed to investigate the effects of intermolecular attractions on the electrostatic behavior of nano-electromechanical systems in purpose of an efficient design [3–7]. Intermolecular attractions such as Casimir [8–11] and van der Waals [12,13] forces are sensible in the modeling of NEMS devices while their effects on MEMS devices are negligible [14].

As a result of the inherent large surface area to volume ratio of nano-scale structures, the crucial effects of the surface energy on their pull-in performance are not negligible. Zhang and Zhao [15] investigated the presence of residual stress in a micro-beam that changes the beam stiffness and thus alter the beam restoring force. The developed linear surface elastic theory by Gurtin and Murdoch [16] and the Young–Laplace equations have been used to illustrate the surface effects, including the surface elasticity and residual stress, in continuum modeling of nano-materials [17,18]. The static and dynamic behavior of beam-type nano-structures incorporating surface effects was studied by He and Lilley [19] and Wang and Feng [20], respectively. Fu and Zhang [21] applied a modified continuum model to study the effect of surface energy on the instability of double-clamped nano-beams. Ansari et al. [22] investigated the size-dependent pull-in instability of nonlinear rectangular nano-plates considering surface stress influences. They studied the electrostatic pull-in instability of circular nano-plates including surface effects [23]. In all of these investigations, the intermolecular attractions as well as the fringing field [24] effects have been neglected.

As a result of the inherent nonlinear behavior of nano-devices due to the electrostatic and intermolecular forces which are inversely proportional to higher orders of the gap between electrodes, finding an exact closed-form analytical solution of the governing equations is not readily achievable. Several approximate analytical methods have been proposed to solve the nonlinear boundary value problems accordingly. In order to treat the nonlinearity of the governing equations, Noghrehabadi et al. [25], Duan et al. [5] and Farrokhabadi et al. [7] used the Adomian Decomposition Method (ADM), which has been discussed widely by Rach [26] and Duan et al. [27]. Ramezani et al. [28] also proposed a second-order polynomial shape function for transverse deflection in their analytical approximate solution by using the Green's function. The proposed Homotopy perturbation method (HPM) [29] as well as reduced-order method by Nayfeh et al. [30] are the other approaches which applied to calculate the analytical expressions for the pull-in instability parameters. Duan and Rach [31] considered a parametric model for a beam-type actuator based on the Euler–Bernoulli beam theory and derived an equivalent integral equation according to the proposed systematic method of Duan–Rach [32]. In all of the aforementioned research, the authors have relied upon the Euler–Bernoulli beam theory to investigate the influences of the surface energy on the instability behavior of NEMS actuator. We note that the influence of the surface energy on the boundary conditions of nano-structures has been ignored by the all mentioned researchers.

In the present study, the authors consider the surface effects on a cantilever NEMS actuator considering the Casimir force, fringing field and surface energy effects using the minimum potential energy principle to obtain the governing equation and to modify the boundary conditions. Using the variational method, the surface energy parameter appears in the boundary condition at the free end which has been previously neglected. Utilizing the Duan–Rach method of determined coefficients (MDC) presented in Duan and Rach [31] to deduce an analytical expression, the pull-in voltage as well as deflection at the beam tip will be obtained by using a second-order polynomial assumption [28]. The obtained results will be compared with the previous available results obtained by Duan and Rach [31] to discuss the effects of the surface energy on the nano-structure's pull-in response considering the modified and prior boundary conditions.

2. The governing equation

A schematic configuration of a cantilever NEMS actuator, with the length L and a uniform rectangular cross section of width b and thickness h, is illustrated in Fig. 1. To obtain the governing equation of the actuator, the principle of minimum potential energy implies that the equilibrium state is achieved by minimizing the free energies. As a result, we need to determine the objective function (deflection) that minimizes the functional (total potential energy).

2.1. Bulk strain energy

The strain energy of the structure can be defined as

$$U_b = \frac{1}{2}\int_0^L \int_A (\sigma_{ij}\varepsilon_{ij})\mathrm{d}A\mathrm{d}x, \tag{1}$$

where σ_{ij} and ε_{ij} are the stress and strain tensors, respectively. By assuming the small deformation, the displacement and strain fields of the beam are

$$u_x = -z\frac{\partial w}{\partial x},\ u_y = 0,\ u_z = w, \tag{2}$$

$$\varepsilon_{xx} = -z\frac{\partial^2 w}{\partial x^2},\ \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_{xy} = \varepsilon_{xz} = \varepsilon_{yz} = 0. \tag{3}$$

Substituting the strain and stress tensors according to Hook's law, the bulk strain energy for the beam can be obtained as

$$U_b = \frac{1}{2}\int_0^L \int_A Ez^2\left(\frac{\partial^2 w(x)}{\partial x^2}\right)^2 \mathrm{d}A\mathrm{d}x, \tag{4}$$

where E is the Young's modulus and z is the distance from the neutral axes of the beam.

2.2. Surface strain energy

The strain energy for the surface layer can be written as

$$U_s = \frac{1}{2}\int_0^L \oint_{\partial A} (\tau_{ij}\varepsilon_{ij})\mathrm{d}s\mathrm{d}x. \tag{5}$$

Having the surface components which are suggested by the continuum theory of Gurtin and Murdoch [16] and Gurtin and Murdoch [33], the strain energy of the surface layer by considering both residual stress and surface elasticity is

$$U_s = \frac{1}{2}\int_0^L \left[E_s I_s\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2b\tau_0\left(\frac{\partial w}{\partial x}\right)^2\right]\mathrm{d}x\,, \tag{6}$$

where E_s and τ_0 are the surface elastic modulus and the surface residual stress, respectively, and $I_s = \int z^2 \mathrm{d}s$.

Fig. 1. Schematic cantilever beam, dielectric spacer, and fixed ground plane.